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ABSTRACT

There is a widespread consensus that China's growth paradigm needs a rebalancing away from investment and external demand and towards consumption and domestic demand. This rebalancing process is supposed to be accompanied by the transition towards Renminbi's full convertibility. In contrast, it is controversial to what extent this adjustment will accelerate the slowdown of China's growth, which will likely occur because of other structural factors. We address these issues by means of a two-country two-stage (before and after Renminbi's full convertibility) model, which reproduces some qualitative features of China's growth pattern and its relationship with the US. We analyze to what extent altering the Chinese exchange rate policy, as well as other structural and policy variables, may have (short-, medium- and long-term) effects on the evolution of the Chinese economy. The paper shows that by lifting the controls on the capital account and letting the currency float, the Chinese authorities will not only expose the economy to the risks of free capital mobility, but will also renounce to important policy instruments for controlling the dynamics of China's economy and the allocation of the national resources.

Key words: Growth-rebalancing; global imbalances; currency convertibility; Chinese economy.

JEL Classification: E42, F33, F41, F43, O41

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1. INTRODUCTION

Many observers have expressed concerns regarding the consequences on capital flows of the Chinese authorities abandoning capital controls and the exchange rate management: China, they argue, may start observing large and destabilizing (“hot money”) short-term capital flows.¹ As we endeavor to make clear with this model, by lifting the controls on the capital account and by letting the currency float, the Chinese authorities do not only face such risks, but also renounce with certainty to a policy mix that proved to be essential for the control of the short -and medium-term dynamics of the Chinese economy. By showing why this is the case, this paper informs the debate on the global and the Chinese rebalancing processes.

In the literature and in the specialized press, the advocated appreciation of the renminbi is often de-contextualized, as if one could be oblivious of the capital account and exchange rate management regimes in which such appreciation occurs. In fact, it is not the value of the nominal exchange rate alone that allowed China to grow fast while preserving its internal stability.² Rather, it is a composite policy mix (i.e., capital restrictions, exchange rate peg, reserve sterilization, independent monetary and fiscal policies) that made such achievements possible, in line with the evidence (Paitnik et al. 2011, Bayoumi and Saborowski 2012) that China has deliberately opted for an original “middle ground” (Aizenman and Sengupta 2013) configuration of the international financial trilemma (Obstfeld et al. 2010). Accordingly, by lifting capital account controls and by letting the currency float, China does not only make an internal rebalancing more likely, but it also relinquishes a combination of policy tools that has empowered the central authorities and allowed them to control the dynamics of the economy. It is apparent that this observation has relevant implications for an emerging country whose economic transformation will be important for, and not independent from, political and institutional changes (Lindbeck 2008).

More precisely, in this paper we develop a two-country two-stage growth model to evaluate the impact of alternative exchange-rate regimes and policy options on the Sino-American relationship and the

¹ Capital inflows may increase if markets expect a further appreciation of the renminbi whereas capital outflows may dominate in the presence of capital reversals or if part of the large domestic savings is abruptly channeled abroad towards higher return investment opportunities and greater diversification.

² Even though Bénassy-Quéré et al. (2013) focus on the effects of Chinese rebalancing on global rebalancing rather than on China’s growth, they also reckon that it is the interaction between the exchange rate regime, capital controls, reserve accumulation and other structural changes that matters the most. For instance, they find that ‘a fall in China’s saving rate would contribute to global rebalancing whatever the exchange rate regime, provided international capital flows do react to interest rate differentials’ and that ‘a monetary reform in itself is unable to rebalance the Chinese economy unless it is accompanied by a shift in government policy concerning net foreign asset accumulation’ (p. 3).

growth-rebalancing process.³ Phase 1 is characterized by an interaction similar to the one identified by Dooley and co-authors (2003) and it reproduces several qualitative aspects of the co-dependency between the US and China. Phase 2 starts when the authorities fully liberalize the capital account and let the exchange rate float.

Three main findings deserve to be previewed at this stage. First, we show that the asymptotic growth rate of the Chinese economy is not affected by the choice of the capital account and exchange rate regime. Second, we show that China may possibly grow at two different asymptotic rates, both independent of the adopted exchange-rate regime. The faster rate is associated with an equilibrium path along which the entire workforce is employed in the market sectors of the economy, whereas the slower rate is associated with an equilibrium path along which some Chinese labor remains involved in rural activities and receives income transfers from the government. Both equilibria are equally possible and compatible with the same parameters of the model: which of the two equilibria emerges depends on whether the Chinese policy-makers, in order to improve the conditions in the rural areas will eventually accept lower rates of growth and will manage to keep the economy along a path characterized by generous transfers in favor of the rural areas.⁴ Third, we show that the capital account and exchange rate regimes impact on the dynamics of the economy in the short- and medium-term even though they do not affect the asymptotic rates of growth. The choice of gradually appreciating the currency and even more the decision to float the exchange rate have direct effects on both the current account balance and on the sectoral composition of employment, which in turn influences the rate of growth during the transitional trajectory. This finding helps to understand the rationale of the export-led growth model embraced since the late 1990s, as well as the Chinese authorities' determination to preserve it in the face of growing internal costs and foreign complaints.⁵

We shall discuss in the next section what motivates this paper in the light of the relevant literature on the Chinese economy; it is worth noticing here that this paper is clearly related to several other strands

³ This work borrows heavily, in terms of modelling strategy, from previous works of ours, in particular Bonatti and Fracasso (2012).

⁴ The central authorities may intend to limit the congestion of urban areas and prevent the abandonment of activities that are vital for the maintenance of the rural territory. Transfers towards the rural areas may also be motivated by social stability concerns as people in these areas, also because of fiscal decentralization and escalating medical costs, tend to enjoy lower social security benefits than urban residents (Chou and Zhang 2009). In 2003 the Chinese authorities introduced a pilot health insurance program, extended to almost all rural areas by 2010, called the New Cooperative Medical System (NCMS). This is a subsidized public health voluntary insurance scheme designed for rural households on a county-by-county base. The wide variation in benefit package and coverage levels across counties, as well as other selection problems in participation, suggest further changes will be implemented in the future (more in Bai and Wu 2013, Chen and Jin 2012, Liu et al. 2011, Wagstaff et al. 2009b).

⁵ On the political economy of 'currency manipulation' bashing in the U.S. Congress, see Ramirez (2012).

of the recent literature on international macroeconomics not directly focusing on China, such as the papers dealing with exchange rate undervaluation, mercantilism and growth (Aizenman and Lee 2007, Levy-Yeyati and Sturzenegger 2010, Razmi et al. 2012, Rodrik 2008), with foreign asset accumulation and capital account policies (Bacchetta et al. 2013, Bayoumi and Saborowski 2012, Benigno and Fornaro 2012, Cheng 2013, Jeanne and Rancière 2011, Jeanne 2012), and with global imbalances (Caballero et al. 2008, Dooley et al. 2003, Mendoza et al. 2009). Our approach differs from the various models developed in these strands of the literature mainly because it is the only one, to best of our knowledge, that adopts a two-country set up, embeds four market sectors (tradable, nontradable and nonmarket activities) so as to capture structural change and the mobilization of surplus labor, postulates GDP growth maximization—rather than welfare maximization—objectives (which in turn allows for persistent current account imbalances and foreign asset accumulation), and provides an orthodox mechanism through which the nominal exchange rate affects the real exchange rate in the absence of normal price stickiness.⁶

The remainder of the paper proceeds as follows. Section 2 motivates the paper by discussing the relevant literature. The building blocks of the model are discussed in section 3, while the implications of alternative exchange rate regimes are presented in section 4. Section 5 is dedicated to the (short-, medium- and long-term) dynamics of China's economy. Section 6 concludes. The mathematical derivations are contained in the Appendix.

2. MOTIVATIONS AND RELEVANT LITERATURE

Since the late 1990s, China has recorded large current account surpluses, in particular towards the United States. The literature on the determinants (and implications) of such state of affairs is vast and covers several economic, political and social aspects. This work embraces the viewpoint proposed by Dooley et al. (2003, 2009) according to which China has purposefully maintained an undervalued exchange rate and kept domestic consumption low in order to promote the growth of its exporting sectors and of the whole economy. This outward orientation accelerated investment and capital accumulation and facilitated the mobilization of the surplus (under-employed) labor in rural areas into the highly productive sectors of the economy (see, among others, Bonatti and Fracasso 2013a,b, Dorucci et al. 2013, Knight and Ding 2012,

⁶ A discussion of other papers that share only some of these features is offered in Bonatti e Fracasso (2013b).

McKinnon and Schnabl 2012, Yang 2012).⁷ The preservation of the international price competitiveness of the Chinese goods, guaranteed by a long lasting undervaluation of the exchange rate in a ‘semi-open’ economy (Bacchetta et al. 2013) where almost only the official authorities can trade foreign financial assets, has been conducive to the accumulation of massive foreign reserves (see Aizenman and Lee 2007, Aizenman and Sengupta 2013, Bonatti and Fracasso 2013b).⁸

The management of the nominal exchange rate would not have produced the expected results in terms of growth, had China not been able to control money supply and domestic inflation through both market-based tools and administrative measures able to limit capital inflows and to sterilize rapidly growing foreign reserves (Aizenman and Sengupta 2013, Bayoumi and Saborowski 2012, Greenwood 2008, McKinnon and Schnabl 2009). Without reserve sterilization, inflation would have grown fast, thereby undoing the authorities’ efforts to control the nominal exchange rate and jeopardizing social stability. Without capital controls, instead, China would have received large amounts of “hot money” capital inflows, led by the self-fulfilling expectation of an (eventual) appreciation of the renminbi.

The global financial crisis has strengthened the position of those encouraging the reduction of global imbalances by means of coordinated interventions, in particular in China and in the United States (Obstfeld 2012, Spence 2013a). An internal rebalancing process, however, has been advocated on different grounds for the Chinese economy, which would benefit from moving away from external demand and investment and toward domestic demand (World Bank 2013).⁹

We succinctly present the three main reasons for an internal rebalancing process in what follows. First, given the current size of the Chinese economy and the global deleveraging process started in the late 2000s, it is argued that the export-led growth paradigm may eventually fail to sustain the growth of GDP in China. Second, the recent attempt by the Chinese authorities to support growth after the collapse of global trade through local governments’ investment projects has shown serious limits (Lee et al. 2013, Lu and Sun 2013, Pettis 2012): the debt ascribable to local public authorities (mainly through local

⁷ The rapid expansion of the production in these sectors has been associated with an intense urbanization process (Dekle and Vandenbroucke 2010, Ding and Knight 2009, Yao and Zhou 2011).

⁸ The accumulation of foreign liquid assets under a regime of capital controls and limited convertibility can also be linked with the intermediation role played by the central bank during the catching up process in a context of financial constraints and domestic financial underdevelopment (Cheng 2013). It may also be related to precautionary motives in a context of limited and intermittent access to international credit markets (Bacchetta et al. 2013, Benigno and Fornaro 2012, Dominguez et al. 2012, Durdu et al. 2009, Jeanne and Rancière 2011).

⁹ See, among the several contributions on the issue, Blanchard and Giavazzi (2006), Dorucci et al. (2013), Knight and Wang (2011), Ma et al. (2013), Prasad (2009), Yang (2012) and Zheng et al. (2009).

financing vehicles and government back institutions) has grown large¹⁰; credit has expanded at extremely high rates (30% per year since the crisis); the returns from investment have fallen, and non-performing loans risk suffocating banks' financial conditions (Pettis 2012, Nabar and N'Diaye 2013). The efforts of the authorities to offset the negative effects of global growth slowdown have thus aggravated various distortions affecting the Chinese economy.¹¹ Third, a series of social and environmental problems risks denting the well-being of the population to the point that the Chinese authorities have already acknowledged the necessity of taking measures even at the cost of accepting lower rates of growth.¹² Indeed, in recent years the central and local authorities have attributed an increasing attention to the conditions of the rural areas (where both State Owned Enterprises (SOE) and foreign-participated enterprises hardly locate) and started undertaking more incisive redistributive actions.¹³

Although necessary to ensure growth in the long run, a correction in the Chinese policy mix will have a negative impact on the country's growth prospects during the transition period: an abrupt interruption of the export-led growth process may severely decelerate the catching-up process of the country in the medium term (Dorucci et al. 2013).¹⁴ Knight (2013a) ranks insufficient growth as one of the major determinants of social instability as both expected future income and expected future income changes have a major impact on individual current well-being and life satisfaction. Furthermore, growth deceleration risks adding to the difficulties that the policy-makers will encounter while trying to engineer a growth regime switch that will negatively affect bureaucrats and local politicians (Xu 2011), as well as a number of other vested interests, especially linked with the SOE that largely have benefited from the

¹⁰ The most recent official audit launched by the National Audit Office (released in December 2013) estimated the local debt reached Rmb17.9tn (\$2.95tn) by the end of June 2013, up from Rmb10.7tn at the end of 2010. This amounts to circa 30% of the Chinese GDP, against the 25% in 2010. This implies that the total public debt is between 50% and 55% of the GDP.

¹¹ A synthetic exposition of the cost distortions characterizing factor markets in China is offered by Brandt et al. (2013), Huang and Wang (2010), Lin (2012), and McKinnon and Schnabl (2009). Based on a cross-country study, Ito and Volz (2013) estimate the impact of various reforms on saving, investment and current account balances in China. The connection of financial distortions, see also Allen et al. (2012), with the recent debt bubble and overinvestment in China is discussed in Pettis (2012). Hang (2012) illustrates how rural reforms in the 1990s re-introduced the distortions lifted during the reforms in the 1970s and 1980s, especially through the penalization of the rural credit cooperatives created by the central government in the early 1980s.

¹² For instance, the rapid process of urbanization has been accompanied by a non-negligible depopulation of some rural areas (Wang and Wan 2014), a (socially and individually) costly mass-migration process, and a worrying increase in (regional and individual) income inequalities. Life satisfaction, especially among the lowest socioeconomic groups has failed to raise with average income (see, among others, Easterlin et al 2012, Knight 2013b). We refer to Knight (2013a) for a discussion of the relationship between growth, life satisfaction, and social instability in China.

¹³ Barone et al. (2013) analyse the spatial features of the Chinese development process and illustrate inequalities across macro-regions and between rural and urban areas.

¹⁴ Bonatti and Fracasso (2010,2012), McKinnon (2006), Rodrik (2010) discuss at length this problem. Chen and Duo (2011) find empirical evidence that Chinese employment (in the tradable and non-tradable sectors) contracts following a real appreciation of the currency, thereby corroborating the existence of short- and medium-term costs associated with rebalancing. For similar reasons Lin (2012) warns about the risks of prematurely shifting from a comparative-advantage-favorable growth paradigm to a comparative-advantage-defying model.

current arrangement.¹⁵ Not to succumb to such private interests, the central authorities need to ensure widespread improvements to the citizens' welfare that compensate the lower growth trajectory during the rebalancing process (see for instance Dorucci et al. 2013, Pilling 2012, Riedel 2011).¹⁶ Finally, this growth deceleration process will occur while other challenges, such as population aging, will also contribute to reduce the growth rate of the economy and to erode its competitive advantage.¹⁷ In sum, on the one hand rebalancing seems necessary for China to escape the co-called 'middle income trap' (Eichengreen et al. 2012) and, on the other hand, it risks being conducive to a medium-term growth deceleration that hinders structural changes and reforms.

For all these reasons, it is of utmost importance to study the transition towards the removal of capital controls and the introduction of full convertibility of the renminbi, as well as to clarify the impact of the various regimes on growth over the long, medium and short term. With the model developed in this paper, we show that the undervaluation of the exchange rate and the procrastination of the full convertibility of the renminbi affect the medium-term trajectory and increase the speed of the Chinese economy during the transition towards the long-term equilibrium. While the model confirms that the undervaluation of the exchange rate does not propel China's growth in the very long-term, it shows that China has a convenience to preserve its growth paradigm as long as possible in the medium term so as to promote its tradable sector. Indeed, as argued by Rodrik (2008, 2010), the expansion of the tradable sector (especially in a developing country) is associated with growth externalities that the authorities may be tempted to seize through either a depreciated real exchange rate (and capital controls), or a

¹⁵ SOE have benefited of large subsidies and a massive indirect wealth transfer from households, which have i) been inadequately compensated for land sales (and seizures) for new property development, ii) enjoyed wage growing less than productivity and iii) received low returns on their bank savings (Yang 2012). Recently, Minxin Pei depicted China as 'gigantic rent-distributing mechanism' where 'the ruling elites have learned to live with each other [...] by carving up the spoil of economic development.' (Pei 2013) In his view this complicates the medium-term adjustment towards a slower growth trajectory. Although less pessimistic, Michael Spence also called for profound and far reaching reforms in the fiscal, regulatory, social security and innovation-related realms, thereby advocating for a process that will occur several years and require political resolve (Spence 2013b).

¹⁶ Central authorities' control over the implementation of policies in the local areas is modest. Prominent explanations China failure to fight bad implementation of national measures focus on corruption, lack of central commitment, resistance from local officials, and a governing system (the "rule of mandates" system, which according to Birney 2014 is opposed to the traditional rule of law system) making it difficult to distinguish proper misdoings from applications of party "mandates" based on relative prioritization of the objectives (and focused on outcomes rather than processes).

¹⁷ The U.N. population projections (UN 2013) suggest that working-age population will start shrinking after 2015 both in absolute and relative terms. Cai and Lu (2013) estimate, for the 13th Five-year Plan period (2016–2020), a reduction in the potential GDP growth rate of about 3% due to population dynamics. See also, among others, CDRF (2011), and Li et al (2012). On whether China has already reached or when it will reach the Lewis turning point, see Das and N'Diaye (2013), Golley and Meng (2011), Knight et al. (2011).

composition of public expenditure biased towards the tradables, or the concession of subsidies to the producers of tradables.¹⁸

3. THE MODEL

The world economy in this model includes two countries, US and China.¹⁹ Three market goods are produced: an internationally tradable good produced in both countries, an (internationally) nontradable good produced and sold only in the US, and an (internationally) nontradable good produced and sold only in China. Hence, in both countries there are firms specialized in the production of tradable goods and firms specialized in the production of nontradable goods. Both the tradable and the nontradable goods can be consumed, while only the tradable good is used as capital in the production of both goods.²⁰ Each country has its own government sector.

Labor is internationally immobile but can freely move across sectors within each country.²¹ In the US, labor that is not employed in the two market sectors receives an unemployment benefit paid by the government. In China, labor that is not employed in the two market sectors is employed in the non-market sector of the economy, consisting of low-productive activities that people undertake if they cannot be employed profitably in the market economy. This reflects what happens, for instance, in the rural areas in China.

Goods and labor markets are perfectly competitive. Both countries are populated by households that supply labor, buy the consumer goods, accumulate financial assets and hold money. Two policy regimes governing the world financial markets are considered. Under the first regime, the Chinese authorities fix the nominal exchange rate and only official transactions in financial assets are permitted because of strict capital controls. Consistently with their public announcements and decisions, the Chinese authorities are assumed to aim at accelerating GDP growth and structural change: notwithstanding its drawbacks in

¹⁸ In a similar spirit, Benigno and Fornaro (2012) rationalize reserve accumulation and trade surpluses with a model including input-related knowledge spillovers in the tradable sector. Our model differs in that we link tradable-related spillovers to capital accumulation so as to capture massive investment in China. This helps to explain why Benigno and Fornaro (2012) conclude that the authorities in emerging markets find reserve accumulation convenient more for its precautionary effects than for its impact on output and productivity growth, whereas we see reserve accumulation as part and parcel of the Chinese growth paradigm.

¹⁹ We draw on the model in Bonatti and Fracasso (2012) to which we refer for a detailed discussion of its building blocks.

²⁰ Although there is no agreed conclusion on the share of tradables and nontradables in total investment (see Turnovsky 1997 and Bems 2008 for, respectively, theoretical and empirical considerations). Our extreme assumption that investment consists only of tradable goods helps us to simplify the set-up and to create a clearer channel for tradable-induced productivity externalities, as suggested by Rodrik (2008).

²¹ The distinction between two main sectors (tradables and nontradables) and the different assumptions regarding labor and capital mobility across sectors and countries are consistent with the standard trade model developed by Obstfeld and Rogoff (1996), Chapter 4. We add to that as we introduce a technological spillover in both sectors, thereby replacing the assumption of exogenous productivity improvements and generating endogenous growth.

terms of household consumption and sterilization costs, this policy fastens the catching-up and enhances the international status of China. The second regime is implemented if the Chinese authorities decide to liberalize the capital account and to let the nominal exchange rate float.

Finally, time is discrete and the time horizon is infinite. There is no source of random disturbances and agents' expectations are rational (in the sense that they are consistent with the true processes followed by the relevant variables), thus implying perfect foresight.

3.1 Firms producing the (internationally) nontradable good

In each country j , $j=us, ch$, there is a large number (normalized to be one) of identical firms, which—in each period t —produce the nontradable good Y_{jNt} , that must be immediately consumed as not storable. Firms produce Y_{jNt} according to the following technology:

$$Y_{jNt} = A_{jNt} K_{jNt}^{1-\gamma_j} L_{jNt}^{\gamma_j}, 0 < \gamma_j < 1, \quad (1)$$

where K_{jNt} and L_{jNt} are, respectively, the capital stock and the labor input used in country j to produce the (internationally) nontradable market good Y_{jNt} , and A_{jNt} is a variable measuring the state of technology of the firms operating in that sector of country j which produces the (internationally) nontradable good Y_{jNt} . It is assumed that technological progress is labor augmenting; more precisely, A_{jNt} is a positive function of the capital installed in the sector of j which produces Y_{jNt} : $A_{jNt} = K_{jNt}^{\gamma_j}$. This assumption combines the ideas that learning-by-doing works through each firm's capital investment and that productivity gains instantly spill over across all firms (Barro and Sala-i-Martin, 1995).²²

The net profit (cash flow) π_{jNt} of the representative firm producing nontradables is given by:

$$\pi_{jNt} = P_{jNt} Y_{jNt} - W_{jt} L_{jNt} - P_{jTt} I_{jNt}, I_{jNt} \geq 0, \quad (2)$$

where, in country j at time t , P_{jNt} and P_{jTt} are the prices of, respectively, the nontradable good and the tradable good, W_{jt} is the nominal wage, and I_{jNt} is capital investment by the representative firm producing nontradables.

The capital stock installed in the nontradable sector evolves according to

$$K_{jNt+1} = I_{jNt} + (1-\delta_j) K_{jNt}, 0 \leq \delta_j \leq 1, K_{jN0} \text{ given.} \quad (3)$$

²² Each single firm takes A_{jNt} as given because its decisions have only a negligible impact on the aggregate stock of capital of the nontradable sector (Frankel 1962). Technological progress is endogenous to the economy, although it is an unintended by-products of firms' capital investment rather than the result of purposive R&D efforts.

In each t , the firms decide on $\{L_{jNt+n}\}_{n=0}^{\infty}$ and $\{I_{jNt+n}\}_{n=0}^{\infty}$ subject to (3) in order to maximize their discounted sequence of net profits

$$\sum_{v=0}^{\infty} \pi_{jNt+v} / \prod_{s=1}^v (1+i_{jt+s}), \quad (4)$$

where $\prod_{s=1}^0 (1+i_{jt+s}) = 1$, and i_{jt} is the nominal interest rate in country j at t .

3.2 Firms producing the (internationally) tradable good

In each country j , there is a large number (normalized to be one) of identical firms producing the tradable good Y_{jTt} according to the following technology:

$$Y_{jTt} = A_{jTt} K_{jTt}^{1-\alpha_j} L_{jTt}^{\alpha_j}, 0 < \alpha_j < 1, \quad (5)$$

where K_{jTt} , L_{jTt} , and A_{jTt} are, respectively, the capital stock, the labor input and the state of technology in country j at time t for the production of the (internationally) tradable good. A_{jTt} is a positive function of the capital installed in the tradable sector: hence, $A_{jTt} = K_{jTt}^{\alpha_j}$.

The net profit π_{jTt} of the representative firm producing tradables is given by

$$\pi_{jTt} = P_{jTt} Y_{jTt} - W_{jt} L_{jTt} - P_{jTt} I_{jTt}, \quad I_{jTt} \geq 0, \quad (6)$$

where I_{jTt} is the capital investment by the firm producing tradables in country j at time t .

The capital stock installed in the tradable sector evolves according to

$$K_{jTt+1} = I_{jTt} + (1-\delta_j) K_{jTt}, \quad 0 \leq \delta_j \leq 1, \quad K_{jT0} \text{ given.} \quad (7)$$

In each t , firms decide on $\{L_{jTt+n}\}_{n=0}^{\infty}$ and $\{I_{jTt+n}\}_{n=0}^{\infty}$ subject to (7) in order to maximize their discounted sequence of net profits

$$\sum_{v=0}^{\infty} \pi_{jTt+v} / \prod_{s=1}^v (1+i_{jt+s}). \quad (8)$$

3.3 Households

The large number of households living in country j is normalized to one. Households are infinitely lived. Consumption, real money balances providing liquidity services and a public good provided by the government enter the period utility function of the representative household of country j , that is u_{jt} :

$$u_{jt} = \ln(C_{jt}) + \chi_j \ln(M_{jt}/P_{jt}) + v(G_{jt}), \quad \chi_j > 0, v' > 0, \quad (9)$$

where, in country j at time t , M_{jt} is the household's nominal money holdings, P_{jt} is the consumer price index, C_{jt} is the household's consumption index, and G_{jt} is the amount of public good provided by the government. The consumption index C_{jt} , which can be interpreted as a composite good, is defined as

$$C_{jt} = C_{jNt}^{\eta_j} C_{jTt}^{1-\eta_j}, \quad 0 < \eta_j < 1, \quad (10)$$

where C_{jNt} and C_{jTt} are the consumption of nontradables and of tradables by the representative household. Given P_{jNt} , P_{jTt} and (10), the consumer price index P_{jt} is

$$P_{jt} = \frac{P_{jNt}^{\eta_j} P_{jTt}^{1-\eta_j}}{D_j}, \quad D_j \equiv \eta_j^{\eta_j} (1-\eta_j)^{1-\eta_j}. \quad (11)$$

The representative household's period budget constraint in country j at time t is:

$$B_{jHt+1} + E_{jt} F_{jHt+1} + M_{jt} + P_{jNt} C_{jNt} + P_{jTt} C_{jTt} \leq (1+i_{jt}) B_{jHt} + E_{jt} (1+i_{it}) F_{jHt} + M_{jt-1} + \pi_{jNt} + \pi_{jTt} \\ + L_{jt} W_{jt} + (H_j - L_{jt}) S_{jt} - T_{jt}, \quad B_{jH0}, F_{jH0} \text{ and } M_{j-1} \text{ given, } i \neq j, \quad (12)$$

where B_{jHt} are the domestic financial assets accumulated during period $t-1$ by the representative household and carried over into period t with nominal yield i_{jt} , E_{jt} ($E_{jt}=1/E_{it}$) is the nominal exchange rate of country j (the price in units of the j -country's currency of one unit of the i -country currency at time t), F_{jHt} are the foreign financial assets (denominated in foreign currency) accumulated during period $t-1$ by the representative household of country j and carried over into period t with nominal yield i_{it} , L_{jt} are the units of labor worked by the household and H_j is the fixed time endowment. S_{jt} is a benefit paid by the domestic government to labor that is not employed in the market sectors of the economy, while T_{jt} are the net monetary transfers ("net taxes") from the representative household to its government. In each period, the representative household is entitled to receive the net profits earned by the firms located in its own country as dividend payments. Nominal balances (no-interest bearing financial assets) M_{jt} are accumulated during period t and carried over into period $t+1$ because of the liquidity services that they provide to the households.

To rule out the possibility that households borrow arbitrary large sums, we impose the usual no-Ponzi condition:

$$\sum_{v=0}^{\infty} \frac{(E_{jt+v} - E_{jt+v+1}) F_{jHt+v+1} + P_{jNt+v} C_{jNt+v} + P_{jTt+v} C_{jTt+v}}{\prod_{s=0}^v (1+i_{jt+s})} \leq B_{jHt} + \quad (13)$$

$$\begin{aligned}
& + E_{jt} F_{jHt} + M_{jt-1} + \sum_{v=0}^{\infty} \frac{\pi_{jNt+v} + \pi_{jTt+v} - T_{jt} + W_{jt+v} L_{jt+v}}{\prod_{s=0}^v (1 + i_{jt+s})} + \\
& + \sum_{v=0}^{\infty} \frac{(H_j - L_{jt+v}) S_{jt+v} + (i_{it+v} - i_{jt+v}) E_{jt+v} F_{jHt+v} - i_{jt+v} M_{jt+v-1}}{\prod_{s=0}^v (1 + i_{jt+s})}, i \neq j.
\end{aligned}$$

The amount of labor supplied by the representative household of country j in period t is determined as follows:

$$L_{jt} = \begin{cases} H_j & \text{if } \frac{W_{jt}}{P_{jt}} > V_{jt} \\ 0 & \text{if } \frac{W_{jt}}{P_{jt}} < V_{jt} \\ \text{indeterminate, otherwise,} \end{cases} \quad (14)$$

where V_{jt} is the reservation wage for households located in country j at time t . We assume that V_{jt} is given by

$$V_{jt} = \varphi_j s_{jt}, \quad s_{jt} \equiv \frac{S_{jt}}{P_{jt}}, \quad \varphi_j \geq 1. \quad (15)$$

In (15), the households' reservation wage increases with the government transfers that they would receive if they remained at home rather than working in the market sectors of the economy. In the case of China, this formulation reflects the fact that the minimum wage at which workers are willing to work in a firm located in an urban area increases with the amount of transfers received by those involved in rural activities.²³ In the case of the US, it captures instead the tendency of the minimum wage to increase with the government benefits and entitlements received by those who are not employed in the market sectors of the economy. Moreover, (15) accounts for the possibility that the households would prefer to stay at

²³ There is growing evidence in China (both statistical data and the increase in labor unrest in foreign-owned factories) of an upward trend in the reservation wages affecting the citizens' willingness to migrate from rural areas to industrial zones (Li et al. 2012). This has several causes, among which the improvements in living conditions in the rural areas. This legitimates our choice of linking the reservation wage of the Chinese urban workers to the government transfers in favor of the rural areas. The reform of the Chinese social system started in the late 1980s (after the collapse of community-based health insurance system following the spate of reforms in the 1970s) has been inspired by the principle of transferring welfare-provision obligations from enterprises to social insurance agencies and individuals (see Wagstaff et al. 2009a, and Zhang and Kanbur 2005 for an overview of the situation up to the early 2000s). In the rural areas, also because of combination of fiscal decentralization (Uchimura and Jütting 2009) and regional income inequality (Barone et al. 2013), large out-of-pockets expenditures, low coverage and limited participation in insurance schemes contributed to keep reservation wages low. More recently, the introduction of the public health insurance reforms (NCMS) has improved the living conditions in the rural areas and thus also contributed to raise reservation wages.

home if the income level that they could get doing so were the same as the market wage: $\phi_j > 1$ means that the households incur some disutility by moving from home in order to work in the market sectors of the economy.

In each period t , households located in country j decide on $\{L_{jt+v}^s\}_{v=0}^{\infty}$, $\{B_{jHt+1+v}\}_{v=0}^{\infty}$, $\{F_{jHt+1+v}\}_{v=0}^{\infty}$, $\{M_{jt+v}\}_{v=0}^{\infty}$, $\{C_{jNt+v}\}_{v=0}^{\infty}$ and $\{C_{jTt+v}\}_{v=0}^{\infty}$ subject to (12), (13) and (14) in order to maximize their discounted sequence of utilities

$$\sum_{v=0}^{\infty} \theta_j^v u_{jt+v}, \quad 0 < \theta_j < 1, \quad (16)$$

where θ_j represents the subjective discount factor of country j 's households.

3.4 Government sector

In each period t the government of country j provides the public good G_{jt} combining nontradable and tradable goods according to

$$G_{jt} = \min(G_{jNt}, \zeta_j G_{jTt}), \quad \zeta_j > 0, \quad (17)$$

where G_{jNt} and G_{jTt} are the quantity of nontradables and of tradables that the government of country j buys to produce the public good. Production is efficient and $G_{jNt} = \zeta_j G_{jTt}$. The parameter ζ_j can be interpreted either as a purely technological parameter or as a policy parameter concerning the characteristics of the public good.

Hence, in country j and at time t , the government has to decide the fraction g_{jt} of the country's GDP to be spent for the production of the public good:

$$P_{jNt} G_{jNt} + P_{jTt} G_{jTt} = g_{jt} (P_{jNt} Y_{jNt} + P_{jTt} Y_{jTt}), \quad 0 \leq g_{jt} < 1. \quad (18)$$

In each t , the government of country j must satisfy its period budget constraint:

$$B_{jGt+1} + E_{jt} F_{jGt+1} + (H_j - L_{jt}) S_{jt} + g_{jt} (P_{jNt} Y_{jNt} + P_{jTt} Y_{jTt}) \leq M_{jt} - M_{jt-1} + T_{jt} + (1+i_{jt}) B_{jGt} + E_{jt} (1+i_{it}) F_{jGt}, \quad B_{jG0}, F_{jG0} \text{ and } M_{j-1} \text{ given}, \quad i \neq j, \quad (19)$$

where B_{jGt} are the domestic financial assets accumulated during period $t-1$ by the j -country's government sector and carried over into period t with nominal yield i_{jt} , and F_{jGt} are the foreign financial assets (denominated in foreign currency) accumulated during period $t-1$ by the j -country's government sector and carried over into period t with nominal yield i_{it} .

The transfers (in real terms) in favor of those not employed in the market sectors of the economy evolve according to:

$$s_{jt+1} = s_{jt}(1 + \omega_{jt}), \quad (20)$$

where the government of country j decides on s_{j0} and $\{\omega_{jt}\}_{t=0}^{\infty}$. Equations (15) and (20) make the reservation wages in both countries adjust over time. This simplified representation of the labor market helps to capture in a dynamic framework the existence of realistic frictions on labor reallocation due to the interaction between changes in relative prices and the evolution of the reservation wage over time.

The no-Ponzi condition of the j -country's government sector is

$$\begin{aligned} & \sum_{v=0}^{\infty} \frac{(E_{jt+v} - E_{jt+v+1})F_{jGt+v+1} + g_{jt+v}(P_{jNt+v}Y_{jNt+v} + P_{jTt+v}Y_{jTt+v})}{\prod_{s=0}^v (1 + i_{jt+s})} + \\ & + \sum_{v=0}^{\infty} \frac{(H_j - L_{jt+v})S_{jt+v}}{\prod_{s=0}^v (1 + i_{jt+s})} + M_{jt-1} \leq B_{jGt} + E_{jt}F_{jGt} + \\ & + \sum_{v=0}^{\infty} \frac{i_{jt+v}M_{jt+v-1} + T_{jt+v} + (i_{it+v} - i_{jt+v})E_{jt+v}F_{jt+v}}{\prod_{s=0}^v (1 + i_{jt+s})}, \quad i \neq j. \end{aligned} \quad (21)$$

3.5 Markets equilibrium conditions

Markets for labor and for the nontradable good are purely domestic. In equilibrium, the labor market of country j is characterized either by $W_{jt}/P_{jt} > V_{jt}$ entailing $L_{jt} = L_{jNt} + L_{jTt} = H_j$, or by $L_{jt} = L_{jNt} + L_{jTt} < H_j$ entailing $W_{jt}/P_{jt} = V_{jt}$. Equilibrium in the country j 's market for the nontradable good requires:

$$Y_{jNt} = C_{jNt} + G_{jNt}. \quad (22)$$

The market for the tradable good is internationally integrated. Equilibrium in this market requires:

$$Y_{usTt} + Y_{chTt} = C_{usTt} + C_{chTt} + G_{usTt} + G_{chTt} + I_{usNt} + I_{usTt} + I_{chNt} + I_{chTt}. \quad (23)$$

In this internationally integrated market, the one-price law must hold:

$$P_{jTt} = E_{jt}P_{iTt}, \quad i \neq j, \quad (24)$$

Money market equilibrium in j requires that money supply is equal to money demand:

$$M_{jt}^s = M_{jt}^d. \quad (25)$$

Equilibrium in the world markets for financial assets requires

$$B_{usHt} + B_{usGt} + F_{chHt} + F_{chGt} = 0 \quad (26)$$

and

$$B_{chHt} + B_{chGt} + F_{usHt} + F_{usGt} = 0. \quad (27)$$

3.6 Policy regimes governing the world financial markets

We consider two phases in the history of the world economy. Phase 1 starts at $t=0$ and is characterized by the capital controls imposed by the Chinese authorities in order to keep their currency undervalued. Phase 2 begins at time $t^*>0$. In period t^* , the Chinese authorities fully liberalize the capital account and let the exchange rate float. A standard welfare function for the Chinese authorities is not specified here because policy choices in China are dictated neither by stabilization concerns nor by the objective of maximizing households' utility.²⁴ There is a wide consensus among observers that the main objective of the Chinese authorities has been—at least up to the present—to keep high the rate of GDP growth without exacerbating income inequalities across social groups and regions (see Bonatti and Fracasso 2013a, Lin 2012, and Yao and Zhou 2011).²⁵

In both phases, the authorities in each country decide on public transfers by setting $\{s_{jt}\}_{t=0}^{\infty}$, on fiscal policy by setting $\{g_{jt}\}_{t=0}^{\infty}$, and on money supply by setting the fixed rate of money growth $\bar{\mu}_{us}$, where $\mu_{jt} \equiv (M_{jt+1} - M_{jt})/M_{jt}$.²⁶

In phase 1, the Chinese capital account is not liberalized: the only international transactions in financial assets that take place are those operated by the Chinese authorities, who also decide on $\{E_{cht}\}_{t=0}^{t^*-1}$, where t^* ($t^*>0$) is the period when, after an irreversible regime switch, phase 2 begins.

Notably, the real exchange rate would be unaffected by the nominal one if and only if $\frac{E_{cht}}{M_{cht}}$ moved equiproportionally with M_{us} . This would have been the case had it not been for capital account controls and reserve accumulation (cum sterilization): these have largely allowed the Chinese authorities to determine the time path of M_{cht} and—independently—the time path of E_{cht} , thereby systematically

²⁴ Also Bénassy-Quéré et al. (2013) do not model the authorities' loss function either. Benigno and Fornaro (2012), instead, assume the authorities maximize households' utility, while taking into account trade-related knowledge spillovers.

²⁵ Illustrating how China could aim at having unity of efficiency and equity, former chief economist and senior vice president of the World Bank Justin Yifu Lin stated: 'Efficiency and rapid growth will always remain priorities' (2012, p. 250).

²⁶ The condition $\bar{\mu}_j > \theta_j - 1$ is necessary to ensure that real money holdings in country j increase asymptotically at the same rate as K_{jTt} and K_{jNt} .

affecting the real exchange rate.²⁷ The authorities in China can keep the real exchange rate undervalued and ensure that the Chinese tradables remain relatively cheap with respect to the US ones.²⁸

Consistently, in phase 1, the Chinese authorities let their foreign asset holdings (henceforth, foreign reserves) adjust to accommodate the flows of funds generated by this mix of policies. In other words, phase 1 is characterized by (26), (27),

$$F_{usHt}=F_{usGt}=F_{chHt}=0, \quad 0 \leq t < t^*, \quad (28)$$

and $E_{cht} = \bar{E}_{cht}$, $0 \leq t < t^*$, where

$$\bar{E}_{cht+1} = \bar{E}_{cht}(1 + \varepsilon_t), \quad 0 \leq t < t^*-1, \quad (29)$$

and both \bar{E}_{ch0} and $\{\varepsilon_t\}_{t=0}^{t^*-2}$ are decided autonomously by the Chinese authorities.²⁹ This crawling peg allows the Chinese authorities to let their currency gradually appreciate while preserving the price competitiveness of the Chinese tradables as China reduces its gap relatively to the US in terms of capital per household in the tradable sector.

Equation (28)—together with (26) and (27)—entails $B_{usHt}+B_{usGt}+F_{chGt}=0$ and $B_{chHt}+B_{chGt}=0$, $0 \leq t < t^*$: the Chinese accumulation of foreign reserves is the counterpart of the US negative net foreign asset position, and in phase 1 it is assumed that the Chinese net holdings of domestic assets are equal to zero.³⁰

In phase 1, China's foreign reserves evolve according to

$$F_{chGt+1}-F_{chGt}=i_{ust}F_{chGt}-TA_{ust}, \quad 0 \leq t < t^*, \quad (30)$$

where $TA_{jt} \equiv P_{jt}(Y_{jt}-C_{jt}-G_{jt}-I_{jNt}-I_{jTt})$ is the trade account of country j (denominated in j currency) at time t . By considering (26) and (28), one can rewrite (30) as

²⁷ In very recent years, the theoretical literature investigating the relationship between capital controls, reserve accumulation, real exchange rate, growth and welfare in developing countries has grown rapidly (Bacchetta et al. 2013, Bayoumi and Saborowski 2012, Cheng 2013, Jeanne 2012, Rodrik 2008). However, to the best of our knowledge, our (orthodox) rationalization of the mechanism through which a nominal peg can turn into a real exchange rate peg has not been explicitly modeled before; the relationship between nominal exchange rate, capital controls, sectoral structural change and economic growth remains an original contribution of this and previous works of ours (Bonatti and Fracasso 2012, 2013a, 2013b).

²⁸ Typically, price rigidity is the device introduced to account for the real effects of a nominal peg in the short run. As our focus extends to longer periods of time, we develop a different mechanism.

²⁹ \bar{E}_{ch0} and $\{\varepsilon_t\}_{t=0}^{t^*-2}$ are, respectively, the level of the nominal exchange rate in period 0 and the time profile of the crawl rate of the exchange rate.

³⁰ Typically, the People's Bank of China sterilizes newly added foreign reserves by selling sterilization bills to domestic agents. As a result, the government sector reduces its holdings of domestic assets and private agents increase theirs. For our purposes, modeling the modalities whereby the Chinese central bank controls the supply of money while accumulating foreign reserves is however not necessary because what matters is that an increase in the government sector's holdings of foreign assets has its counterpart in an improvement of the country's trade account.

$B_{usHt+1} + B_{usGt+1} = (1 + i_{ust})(B_{usHt} + B_{usGt}) + TA_{ust}$, that is the consolidated (government and private sector) balance sheet of the US economy under this policy regime. For our purposes, given the Chinese authorities' willingness to accumulate foreign reserves, it is immaterial how the US external debt is divided up according to government and private sector net liabilities.

In period t^* , the Chinese authorities opt for an irreversible regime switch, by liberalizing the capital account and letting the nominal exchange rate float consistently with the two countries' policy choices and market fundamentals.³¹ Under this new regime, the interest-parity condition holds:

$$(1 + i_{cht}) = \frac{E_{cht}}{E_{cht-1}} (1 + i_{ust}), \quad t \geq t^*. \quad (31)$$

In phase 2, the Chinese authorities also decide on the maximum amount of US trade deficit—as a fraction ξ of the US GDP—that they are willing to finance in each period.³² Therefore, China's net foreign asset position (in US currency) evolves according to

$$\begin{aligned} F_{chHt+1} + F_{chGt+1} - E_{ust}(F_{usHt+1} + F_{usGt+1}) - [F_{chHt} + F_{chGt} - E_{ust-1}(F_{usHt} + F_{usGt})] = \\ = i_{ust}[F_{chHt} + F_{chGt} - E_{ust-1}(F_{usHt} + F_{usGt})] - TA_{ust}, \quad t \geq t^*, \quad (32) \end{aligned}$$

where $TA_{ust} \geq -\xi(P_{usNt}Y_{usNt} + P_{usTt}Y_{usTt})$, $\xi \geq 0$, $t \geq t^*$.

Summarizing, phase 2 (for $t \geq t^*$) is characterized by equations (26), (27), (31) and (32). Also in this phase, as in phase 1, the possibility for the US to run a persistent external deficit rests on the Chinese authorities' willingness to finance it.

It should be stressed that the Chinese authorities are free to decide when the regime switch has to take place. It is even possible that the regime switch is postponed forever ($t^* \rightarrow \infty$) so that the Chinese authorities never liberalize the capital account and the nominal exchange rate never floats.

4. IMPLICATIONS OF ALTERNATIVE EXCHANGE-RATE REGIMES

In this section we shall discuss some implications of the two exchange-rate regimes under the realistic hypothesis that, at equilibrium, the US always employs all its labor force in the modern sectors of the economy (see section 1 of the Appendix for the derivation of the equations characterizing an equilibrium

³¹ We evaluate either a 'semi-open' (as defined by Bacchetta et al. 2013) or a fully-open set up. Bénassy-Quéré et al. (2013), instead, look at a continuous range of capital mobility.

³² Among the alternative ways to set a limit to the US external deficit financed by the Chinese authorities, this formulation is chosen for the sake of simplicity and analytical convenience.

path of the economy). Accordingly, at time 0, the initial capital endowments in both market sectors of the economy are relatively large with respect to the reservation wage only in the US.³³

As argued in the introduction, the Chinese authorities set the time profile of the nominal exchange rate, control capital flows and accumulate reserves so as to accelerate economic growth. Under this policy regime, equation (29) can be used to rewrite (A36) as

$$\frac{C_{usTt}}{C_{chTt}} = \bar{E}_{cht} \left[\frac{(1 + \bar{\mu}_{us} - \theta_{us})(1 - \eta_{us})\chi_{ch}(1 + \bar{\mu}_{us})^t M_{us-1}}{(1 + \bar{\mu}_{ch} - \theta_{ch})(1 - \eta_{ch})\chi_{us}(1 + \bar{\mu}_{ch})^t M_{ch-1}} \right], \quad 0 \leq t < t^*. \quad (33)$$

Equation (33) shows that, by keeping their currency undervalued, the authorities in China compress the consumption of tradables relatively to that of the US. Given the existing stocks of capital, the Chinese reservation wage and the other policy variables at any time $t < t^*$, a depreciation of the Chinese nominal exchange rate (a larger \bar{E}_{cht}) brings about, for any level of L_{usTt} , a higher employment level in the Chinese tradable sector (a higher L_{chTt}).³⁴ This is due to the fact that: i) a nominal depreciation in China is matched by a depreciation of the real exchange rate as R_{cht} increases with

\bar{E}_{cht} (where $R_{jt} \equiv \frac{E_{jt}P_{it}}{P_{jt}}$, $i \neq j$)³⁵ and, other things being equal, ii) a more depreciated real exchange rate in China (a higher R_{cht}) tends to be associated with a larger L_{chTt} . One can use (11), (24), (A5),

(A8) and $R_{cht} = \frac{E_{cht}P_{ust}}{P_{cht}}$ to obtain $L_{chTt} = S(R_{cht}, L_{usTt}, N_{cht}, K_{chTt}, K_{chNt}, K_{usTt}, K_{usNt})$, (where

$N_{jt} \equiv \frac{V_{jt}}{K_{jTt}}$, and $S_{R_{cht}} > 0$) which is the relationship linking the real exchange rate to L_{chTt} for $t < t^*$.

³³ Since the possibility for a country to employ all its labor in the two market sectors depends on its endowments of capital in both sectors relatively to its reservation wage (see (A8)), we are assuming that the US initial endowments K_{usT0} and K_{usN0} are relatively large with respect to V_{us0} , while China has smaller initial stocks of capital per household with respect to V_{ch0} and in some period it may employ some of its labor in the traditional sector. It should be noticed that K_{jTt} and K_{jNt} can be considered as, respectively, the stock of capital per household in the tradable sector and the stock of capital per household in the nontradable sector.

³⁴ By applying the implicit function theorem to (33), it can be shown—considering also (A8) and (A27)—that L_{chTt} is an increasing function of \bar{E}_{cht} .

³⁵ One can verify that $\frac{\partial R_{cht}}{\partial \bar{E}_{cht}} > 0$ by considering two conditions. First, thanks to (11), (24), (A5) and (A8), one can

write $R_{cht} = \frac{E_{cht}P_{ust}}{P_{cht}} = R(L_{chTt}, L_{usTt}, K_{chTt}, K_{chNt}, K_{usTt}, K_{usNt})$, where $R_{L_{chTt}} > 0$. Second, a larger \bar{E}_{cht} brings about a higher L_{chTt} (see above).

This is a crucial intermediate result because L_{chTt} is one of the variables in terms of which the equilibrium path of the two-country economy is characterized.

Equation (33) reveals that the compression of the Chinese consumption of tradables is a crucial determinant of the Chinese growth model and not an incidental condition. It affects the relative prices of tradable and nontradables in both countries (thereby influencing the composition of domestic output and the allocation of the inputs) but it also impacts on the investment in the tradable sector. The preservation of an undervalued currency through reserve accumulation and capital controls is therefore conducive to higher capital investment and growth for $t < t^*$.³⁶

By using (A8), (A27) and (A32) for substituting, respectively, L_{jNt} (recalling that $L_{usNt} = H_{us} - L_{usTt} \forall t \geq 0$), C_{jTt} and K_{jNt} , one can also verify that equation (33) defines implicitly the level of employment in the US tradable sector as a function of L_{chTt} , L_{chNt} , $Z_t \equiv \frac{K_{usTt}}{K_{chTt}}$, g_{cht} , g_{ust} , \bar{E}_{cht} , $\bar{\mu}_{us}$ and $\bar{\mu}_{ch}$:

$$L_{usTt} = e(L_{chTt}, L_{chNt}, Z_t, g_{cht}, g_{ust}, \bar{E}_{cht}, \bar{\mu}_{us}, \bar{\mu}_{ch}), \quad e_{\bar{E}_{cht}} < 0, \quad 0 \leq t < t^*. \quad (34)$$

In phase 1, when the Chinese authorities set the nominal exchange rate, monetary policies in both countries can affect the dynamics of the real variables. This is not the case when the nominal exchange rate can float in a way consistent with the countries' policies and market fundamentals. Under this policy regime, which characterizes phase 2, L_{usTt} and L_{chTt} are linked by the following relationship (see section 2 of the Appendix)

$$L_{usTt} = l(L_{chTt}) = \left[\frac{(1 - \alpha_{ch})L_{chTt}^{\alpha_{ch}} + \delta_{us} - \delta_{ch}}{(1 - \alpha_{us})} \right]^{\frac{1}{\alpha_{us}}}, \quad t \geq t^*. \quad (35)$$

In phase 1, the Chinese authorities keep the currency undervalued so as to maintain the Chinese tradables relative cheap with respect to the US tradables. This policy can be modelled in terms of i) a measure of the aggressiveness of the mercantilist strategy at time 0

³⁶ It is well known that relative prices affect the intra- and inter-temporal allocation of productive inputs (see for example Obstfeld and Rogoff, 1996, 2007). What the model highlights are i) the link between nominal peg and relative prices in presence of capital restrictions and accumulation (cum sterilization) of foreign reserves, and ii) the impact of the sectoral allocation of factors on growth in presence of growth externalities.

³⁷ At time 0, the level of employment in the US tradable sector depends also on the initial endowments of capital K_{chT0} , K_{usT0} , K_{chN0} and K_{usN0} .

$$\bar{E}_{ch0} = \frac{Q(1 + \bar{\mu}_{ch} - \theta_{ch})(1 - \eta_{ch})\chi_{us}M_{ch-1}Z_0}{(1 + \bar{\mu}_{us} - \theta_{us})(1 - \eta_{us})\chi_{ch}M_{us-1}}, Q > 0, 0 \leq t < t^*, \quad (36)$$

where Q is a constant (whose value is decided by the Chinese authorities) that measures the degree of “aggressiveness” of the mercantilist strategy³⁸, and ii) the rates of the crawling peg (see equation (29))

$$\varepsilon_t = \frac{(1 + \bar{\mu}_{ch})(1 + \rho_{ust})}{(1 + \bar{\mu}_{us})(1 + \rho_{cht})} - 1, 0 \leq t < t^*. \quad (37)$$

where ρ_{jt} is the rate of growth of capital in the tradable sector of country j . Equation (37) reflects that fact that—given the monetary policies in both countries—the exchange rate in China appreciates whenever the Chinese capital per household in the tradable sector grows faster than its US counterpart, so as to maintain roughly constant its competitive edge with respect to the US.

Given (A2), (33), (36) and (37), one can rewrite equation (34) as

$$L_{usTt} = f(L_{chTt}, L_{chNt}, g_{cht}, g_{ust}, Q), f_Q < 0, f_{g_{ust}} < 0, 0 \leq t < t^*. \quad (38)$$

Using equations (35) (under the floating exchange rate regime) and (38) (under the peg) and defining \underline{Q}_t as the value of Q such that $l(L_{chTt}) = f(L_{chTt}, L_{chNt}, g_{cht}, g_{ust}, Q)$, we say that when $Q > \underline{Q}_t$, the Chinese currency is undervalued. When this occurs, the US employment in the tradable sector is lower than the equilibrium level we would observe in a floating exchange-rate regime. As said, we assume that capital account controls and the accumulation of foreign reserves in China in phase 1 are associated with the undervaluation of the currency : accordingly, we assume that $Q > \underline{Q}_t, 0 \leq t < t^*$.⁴⁰ It is worth recalling that, as can be seen from equations (36)-(37) and (A37), after having decided on the preferred level of Q given the US monetary policy $\bar{\mu}_{us}$, the authorities in China can choose their preferred combination of (equilibrium) level of the nominal interest rate and level (and time profile) of the nominal exchange rate.⁴¹

5. GROWTH DYNAMICS

³⁸ Clearly, a smaller Q implies, other things being equal, a less depreciated China’s nominal and real exchange rates.

³⁹ Again, at time 0, the level of employment in the US tradable sector depends also on the initial endowments of capital K_{chT0} , K_{usT0} , K_{chN0} and K_{usN0} .

⁴⁰ Notably, the aggressiveness of the mercantilist policy (i.e., the level of Q) affects the dynamics of the real variables. In the presence of financial repression in China and, thus, of independent monetary policies, the relative price of the tradables with respect to the nontradables is distorted in both countries. As argued, this affect both the intra- and the inter-temporal allocation of the resources.

⁴¹ Given $\bar{\mu}_{us}$, there is a continuum of combinations of $\bar{\mu}_{ch}$ and \bar{E}_{cht} that are consistent with a given level of Q .

To save space, we examine the dynamics of the world economy only for the case in which the United States is the relatively impatient country ($\theta_{us} < \theta_{ch}$), consistently with the evidence in favor of a lower propensity to save for U.S. households relative to their European and Asian counterparts (see Ghironi et al., 2008).⁴² We shall address in three separate subsections the very long run, the medium run, and the short run.

5.1 The (very) long run

We study (very) long-run growth by analyzing the asymptotic behavior of the two-country economy both in the case in which the Chinese authorities liberalize the capital account letting the nominal exchange rate float at time $t^* < \infty$, and in the case in which they postpone this regime switch forever ($t^* \rightarrow \infty$) (see section 4 of the Appendix). This analysis allows us to state and discuss a few propositions concerning long-run (asymptotic) growth.

Proposition 1. The exchange-rate regime adopted by the Chinese authorities has no effect on China's asymptotic growth.

Proof: See section 4 of the Appendix.

The intuition behind this result is quite straightforward: as China tends to grow at a faster rate than the US (see the Appendix), China's economic growth is increasingly less (more) dependent on foreign (domestic) demand, and its exchange-rate policy becomes asymptotically irrelevant for its growth performance. The same is not true for the US, whose long-run growth tends to decline with the degree of "aggressiveness" of China's exchange-rate policy, i.e., with the level of Q set by the Chinese authorities.

Proposition 2. There are two different asymptotic rates at which the Chinese economy may possibly grow. Neither of them depends on the exchange-rate regime adopted in China. The faster rate is associated with an equilibrium path characterized by "full-employment" (i.e., a path along which the entire Chinese workforce is employed in the market sectors of the economy), while the slower rate is associated with an equilibrium path along which some Chinese labor is not employed in the market sectors of the economy ("underemployment").

Proof: See section 4 of the Appendix.

According to Proposition 2, the structural and policy parameters of the world economy are consistent with two possible long-run equilibrium paths: in the presence of self-fulfilling expectations, both growth paths may materialize. However, the one associated with full-employment is saddle-path

⁴² We refer to section 3 of the Appendix for the systems of difference equations governing the dynamics of the economy in the various phases.

stable, while the other is unstable. This implies that while the former is robust with respect to small perturbations, the latter is such that the economy may be easily moved further away from it when hit by some shocks (and if the policy makers do not promptly intervene). Indeed, in a neighborhood of the asymptotic equilibrium path characterized by underemployment, the economy's rate of growth tends to be equal to the government controlled rate of growth of the per capita real transfers in favor of those who do not work in the modern sectors of the economy.

One may ask what is the purpose of the frequent policy interventions ("fine tuning") necessary to keep the economy along this low-growth equilibrium path. A possible answer is that the policy-makers may become increasingly willing to use government transfers in favor of those involved in rural activities in order to limit the congestion of urban areas where modern production facilities are concentrated, thus avoiding the complete abandonment of activities that maintain the rural territory and preserve the environment.⁴³ This would be in line with the Chinese authorities' repeated statements that they are willing to tackle some social issues associated with the well-being of the population at the cost of reducing the rate of GDP growth. Though urbanization may be an important aspect of the Chinese development, it is the well-being of those lagging behind that influences the feasibility of the growth strategy as it impacts on social and political stability.

Proposition 3. China's asymptotic rate of growth associated with the full-employment equilibrium path increases with the fraction of China's GDP devoted to the provision of the public good, if the latter is produced by using a relatively small proportion of nontradable good. For public expenditure to positively impact on the asymptotic growth rate, thus, ζ_{ch} needs be below a critical threshold $\bar{\zeta}_{ch}$ that, in turn, depends on α_{ch} , γ_{ch} , η_{ch} , θ_{ch} , δ_{ch} and H_{ch} .

Proof: See section 5 of the Appendix.

Along the full-employment equilibrium path, long-run real growth in China is sensitive to both the size of the public expenditures and its composition in terms of tradables and nontradables. Indeed, fiscal policy can affect the composition of aggregate demand, thereby shifting domestic production towards the tradable sectors. Under certain conditions, this shift may favor long-run growth which is positively related to the marginal productivity of capital in the production of tradables. In this perspective, as long as ζ_{ch}

⁴³ According to the National People's Congress, public expenditures in agriculture grew by an annual rate of 23% in the period 2007-2012, in addition to the tripling of subsidies and bank loans, the large number of roads built in rural areas, the infrastructures put in place to guarantee access to safe drinking water to 300 million rural residents.

exceeds the critical threshold $\bar{\zeta}_{ch}$, public expenditures can partially offset the impact of lower exports on growth.

Proposition 4 China's asymptotic rate of growth associated with the full-employment equilibrium path increases with the size of China's population.

Proof: See section 6 of the Appendix.

Proposition 4 captures a scale effect that is common to other endogenous growth model: the abundance of labor can be a factor enhancing long-run growth if it stimulates the investment in those complementary productive assets whose accumulation has positive spillovers on the entire economy. This entails that a reduction of the workforce due to the demographic transition that is currently underway in China will bring about a decrease in the long-run rate of GDP growth; this is a policy-unrelated development that will slow down growth in China notwithstanding other decisions made by the authorities.⁴⁴

5.2 The medium run

We study the medium run by linearizing the system of difference equations governing the equilibrium trajectories around the paths that the economy may approach asymptotically. In particular, we focus on the unique transitional path converging toward the equilibrium path characterized by full-employment, both in the case in which the Chinese authorities let the nominal exchange rate float at time $t^* < \infty$, and in the case in which they postpone this regime switch forever ($t^* \rightarrow \infty$) (see section 7 of the Appendix).

We can establish the following proposition regarding the transitional path:

Proposition 5. Along the unique transitional path converging toward the equilibrium trajectory characterized by full-employment, the employment level of China's tradable sector is higher when the US tends in the long term to run a trade account deficit rather than to have a surplus or a balanced trade account (i.e., L_{chTt} is higher when $TA_{us} < 0$ rather than $TA_{us} \geq 0$).

Proof: See section 7 of the Appendix.

⁴⁴ Both population aging and lower international migration rates tend to reduce the number of people able to work in the market sector of the economy (Cai and Lu 2013). The introduction in 2007 of the Urban Resident Basic Medical Insurance (URBMI), a subsidised voluntary public health insurance scheme for the urban residents without formal employment that complements the Urban Employee Basic Medical Insurance (UEBMI) for the urban employed (Lin et al. 2009, Liu and Zhao 2012), can be interpreted both as a welfare-enhancing provision and as a means to preserving a steady inflow of working-age migrants from the rural areas. This is confirmed by the announced reform of the *hukou* registration system (Cai 2011).

Proposition 5 suggests that the Chinese authorities may continue to finance the US trade deficit also in the future in the attempt to sustain China's production of tradables, whose externalities are key drivers of long-term growth (as discussed in Rodrik (2008) and in the Introduction).

Also the following proposition holds:

Proposition 6. The choice of exchange-rate regime by the Chinese authorities affects the transitional path. In particular, along the unique transitional trajectory converging toward the equilibrium path characterized by full-employment, the exchange-rate policy followed by the Chinese authorities directly affects the employment level of China's tradable sector.

Proof: See section 8 of the Appendix.

Consistently with propositions 5 and 6, the choice of the Chinese authorities either to maintain the peg but appreciate persistently the currency (i.e., a permanent low Q), or—more radically—to abandon the exchange-rate pegging in favor of a floating exchange-rate regime, has a direct impact on both the US trade deficit and the sectoral composition of both countries' total employment, thus influencing their respective rate of growth during the transitional trajectory.

It is not surprising that US trade deficits toward China will be lower if the latter switches to a floating exchange-rate regime rather than if it sticks to a policy of systematic exchange-rate undervaluation. However, it is significant that, by fully liberalizing their capital account and floating their nominal exchange rate, Chinese policy makers will lose an important instrument to directly control the dynamics of China's real economy. In any case, a full appraisal of the implications of such a regime shift for the long-term evolution of the Chinese economy requires an analysis of its short-term effects, since its future consequences depend also on its impact on today's investment decisions. Indeed, the anticipation of a regime shift occurring in the future has some influence on agents' current decisions, which we are going to discuss in the next paragraph.

5.3 The short run

We assess the direction of the short-term effects (at time 0) of the Chinese government's announcement regarding the timing of the full liberalization of the capital account and the floating of the exchange rate, that is regarding t^* . We use a numerical example in order to compare two polar cases, the case in which the government announces that the regime switch will take place in the next period ($t^*=1$) and the case in which it announces that it will never occur ($t^*\rightarrow\infty$) (see section 9 of the Appendix). In this analysis, it is taken for granted that government announcements are considered fully credible by the public, and that all

the other policies (in particular, those implemented at time 0), the structural parameters of the economy and the initial conditions will remain unchanged. It is also worth to mention that in our example we compare equilibrium paths converging asymptotically to the trajectories characterized by full-employment, since the latter are saddle-path stable (differently from those characterized by under-employment), and thus robust to small perturbations and more likely to materialize.

As we know from Proposition 2, under both exchange-rate regimes we have the same asymptotic rate of growth, while—not surprisingly—the US trade deficit is asymptotically higher when $t^* \rightarrow \infty$. Moreover, consistently with Proposition 6, our example shows that—along the transitional path—the workforce employed in the Chinese tradable sector is larger (and the opposite is true for the workforce employed in the non-tradable sector) when $t^* \rightarrow \infty$: the systematic under-evaluation of the exchange rate shrinks that part of the market economy which is not exposed to foreign competition. At time 0, both when $t^*=1$ and when $t^* \rightarrow \infty$, there is a fraction of the Chinese labor employed in the rural sector, but significantly this fraction is higher when $t^* \rightarrow \infty$, although the rate of real GDP growth and capital accumulation in period 0 is higher when the regime switch is postponed forever. In other words, in the initial period it seems to emerge in China a trade-off between employment in the modern sectors of the economy and growth. Notice that this result does not depend on the fact that the tradable sector is possibly less labor intense than the non-tradable sector. In contrast, the explanation of this result lies in the demand for Chinese tradables generated by the investment boom that is going on at time 0 in the US tradable sector when there is certainty that in the next period the Chinese currency will appreciate because of the end of the exchange-rate pegging, in a situation where still China enjoys an advantage in terms of competitiveness thanks to the under-appreciation of its currency.

6. CONCLUSIONS

In this two-country two-stage growth model, we capture the main qualitative features of the Sino-American economic relation and we analyze the implications for the dynamics of the Chinese economy of a change in the policy mix, in terms of capital account and exchange rate regimes, adopted by the Chinese authorities.

The first stage of the model (phase 1) reproduces the Sino-American co-dependency emerged in the last decade (Dooley et al. 2003). The Chinese leadership, thanks to tight controls on the capital account and to reserve sterilization, maintains an undervalued exchange rate *vis-à-vis* the US dollar with a

view to boosting the country's exporting sectors and to mobilizing the surplus labor in the rural areas into the market sectors. This capital account and exchange rate regimes produce persistent current account surpluses and a rapid accumulation of foreign reserves in China. The second stage of the model, i.e. phase 2, starts when the authorities fully liberalize the capital account and let the exchange rate float.

The paper shows that the asymptotic growth rate of the Chinese economy is not affected by the choice of the capital account and exchange rate regime. On the contrary, the undervaluation of the renminbi leads to a faster accumulation of capital in China in the medium term, thereby reducing the time necessary to absorb the Chinese manpower into the modern sectors of the economy. Capital controls and the fixed exchange-rate regime do impact on the dynamics of the economy both in the medium-term and in the short-term: the degree of exchange rate undervaluation and the compression of domestic consumption affect the sectoral composition of employment, which in turn influences the rate of growth of the economy along the transitional path.

These theoretical findings contribute to the literature and the debate on the future of the Chinese economy in two main ways. First, they provide an explanation of why China has maintained an undervalued exchange rate, restricted the capital flows, and accepted a very low domestic consumption despite the growing implementation costs and potential risks of this strategy: such a policy mix was necessary to the medium-term promotion of the exporting sectors, in turn essential to fasten the process of catching-up and labor mobilization. Second, these results show that, as long as they do not liberalize the capital account and float the nominal exchange rate, the Chinese policy-makers retain a combination of policy tools that allows them to control the dynamics of China's real economy in the short and medium terms. Thus, the decision to float the currency and to liberalize the capital account may facilitate the rebalancing of the economy, but also certainly deprives the authorities of a unique composite of tools for intervention and control. This helps to understand the reluctance of the Chinese authorities to abandon the export-led growth strategy and its underpinning policy determinants, notwithstanding the harsh complaints of the U.S. Congress (Ramirez 2012) and the growing costs of reserves sterilization and capital controls.

The model shows that the Chinese economy may alternatively grow at two different asymptotic rates, both independent of the adopted exchange-rate regime. The faster rate is associated with an equilibrium path characterized by the entire Chinese workforce employed in the market sectors of the economy. The slower rate is instead associated with an equilibrium path along which some Chinese labor

remains employed in rural activities and is subsidized by government transfers. This suggests that if the policy-makers will grow concerned with the congestion of urban areas and the abandonment of the activities that maintain the rural territory, they may ultimately choose the second trajectory, despite the direct costs of the transfers in favor of the population located in rural areas and the foregone benefits of a higher GDP growth. This finding is fully consistent with, and sheds light on, the idea that the necessity of addressing some growing social and territorial problems may eventually force the Chinese authorities to revise downwards their growth targets and to undertake more effective redistributive measures.

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APPENDIX

1 Derivation of the equations characterizing an equilibrium path

1.1 By using (5) and (7), one can rewrite (23), i.e., the equilibrium condition of the world market for the tradable good, as

$$\begin{aligned} Z_t \left[\frac{G_{usTt}}{K_{usTt}} - L_{usTt}^{\alpha_{us}} + \left(1 + \frac{K_{usNt+1}}{K_{usTt+1}} \right) (1 + \rho_{ust}) + \frac{C_{usTt}}{K_{usTt}} - (1 - \delta_{us}) \left(1 + \frac{K_{usNt}}{K_{usTt}} \right) \right] + \\ + \frac{G_{chTt}}{K_{chTt}} - L_{chTt}^{\alpha_{ch}} + \left(1 + \frac{K_{chNt+1}}{K_{chTt+1}} \right) (1 + \rho_{cht}) + \frac{C_{chTt}}{K_{chTt}} - (1 - \delta_{ch}) \left(1 + \frac{K_{chNt}}{K_{chTt}} \right) = 0, \\ Z_t \equiv \frac{K_{usTt}}{K_{chTt}}, \quad \rho_{jt} \equiv \frac{K_{jTt+1}}{K_{jTt}} - 1, \end{aligned} \quad (A1)$$

where the law of motion of Z_t is given by

$$Z_{t+1} = \left(\frac{1 + \rho_{ust}}{1 + \rho_{cht}} \right) Z_t. \quad (A2)$$

1.2 From firms' first-order conditions with respect to labor, we get

$$L_{jNt} = \left(\frac{\gamma_j P_{jNt} A_{jNt} K_{jNt}^{1-\gamma_j}}{W_{jt}} \right)^{\frac{1}{1-\gamma_j}}, \quad (A3)$$

$$L_{jTt} = \left(\frac{\alpha_j P_{jTt} A_{jTt} K_{jTt}^{1-\alpha_j}}{W_{jt}} \right)^{\frac{1}{1-\alpha_j}}. \quad (A4)$$

By using (A3) and (A4), one can check that in equilibrium the relative price of the nontradable good in terms of the tradable good must equalize the ratio between the marginal productivity of labor in the production of tradables and the marginal productivity of labor in the production of nontradables:

$$\frac{P_{jNt}}{P_{jTt}} = \frac{\alpha_j K_{jTt} L_{jNt}^{1-\gamma_j}}{\gamma_j K_{jNt} L_{jTt}^{1-\alpha_j}}. \quad (A5)$$

1.3 Consider that (14), (21), (A3) and (A4)—together—rule out the possibility that $\frac{W_{jt}}{P_{jt}} < V_{jt}$ along an

equilibrium path. Hence, labor market equilibrium requires $\frac{W_{jt}}{P_{jt}} \geq V_{jt}$. Furthermore, by inspecting (14)

and (21), one can verify that $\frac{W_{jt}}{P_{jt}} > V_{jt}$ entails $L_{jTt} + L_{jNt} = H_j$. In its turn, this implies (considering (11),

$$(A4), (A5) \text{ and } N_{jt} \equiv \frac{V_{jt}}{K_{jTt}}) \text{ that } \frac{\alpha_j D_j}{L_{jTt}^{1-\alpha_j}} \left[\frac{\gamma_j K_{jNt} L_{jTt}^{1-\alpha_j}}{\alpha_j K_{jTt} (H_j - L_{jTt})^{1-\gamma_j}} \right]^{\eta_j} > N_{jt} \text{ entails } L_{jNt} = H_j - L_{jTt}. \text{ Thus,}$$

$$L_{jNt} = H_j - L_{jTt} \text{ if } \frac{\alpha_j D_j}{L_{jTt}^{1-\alpha_j}} \left[\frac{\gamma_j K_{jNt} L_{jTt}^{1-\alpha_j}}{\alpha_j K_{jTt} (H_j - L_{jTt})^{1-\gamma_j}} \right]^{\eta_j} > N_{jt}. \quad (A6)$$

Finally, $\frac{\alpha_j D_j}{L_{jTt}^{1-\alpha_j}} \left[\frac{\gamma_j K_{jNt} L_{jTt}^{1-\alpha_j}}{\alpha_j K_{jTt} (H_j - L_{jTt})^{1-\gamma_j}} \right]^{\eta_j} \leq N_{jt}$ entails $\frac{W_{jt}}{P_{jt}} = V_{jt}$ (again, consider (11), (14), (21),

(A4) and (A5)). Thus, one can use (A3) and (A5) to obtain

$$L_{jNt} = \left[\left(\frac{\gamma_j K_{jNt}}{\alpha_j K_{jTt}} \right)^{\eta_j} \frac{\alpha_j D_j N_{jt}^{-1}}{L_{jTt}^{(1-\alpha_j)(1-\eta_j)}} \right]^{\frac{1}{(1-\gamma_j)\eta_j}} \text{ if } \frac{\alpha_j D_j}{L_{jTt}^{1-\alpha_j}} \left[\frac{\gamma_j K_{jNt} L_{jTt}^{1-\alpha_j} K_{jTt}^{-1}}{\alpha_j (H_j - L_{jTt})^{1-\gamma_j}} \right]^{\eta_j} \leq N_{jt}. \quad (A7)$$

By putting together (A6) and (A7), one gets:

$$L_{jNt} = \begin{cases} H_j - L_{jTt} & \text{if } \frac{\alpha_j D_j}{L_{jTt}^{1-\alpha_j}} \left[\frac{\gamma_j K_{jNt} L_{jTt}^{1-\alpha_j}}{\alpha_j K_{jTt} (H_j - L_{jTt})^{1-\gamma_j}} \right]^{\eta_j} > N_{jt} \\ L\left(\frac{K_{jNt}}{K_{jTt}}, N_{jt}, L_{jTt}\right) = \left[\left(\frac{\gamma_j K_{jNt}}{\alpha_j K_{jTt}} \right)^{\eta_j} \frac{\alpha_j D_j}{N_{jt} L_{jTt}^{(1-\alpha_j)(1-\eta_j)}} \right]^{\frac{1}{(1-\gamma_j)\eta_j}} & \text{otherwise.} \end{cases} \quad (A8)$$

In (A8), one can check that the possibility for country j to employ all its workforce H_j in the two market sectors of the economy depends crucially on its endowments of capital in both sectors relatively to its reservation wage. The law of motion of N_{jt} , i.e., the ratio in country j between the reservation wage and the capital installed in the tradable sector is given by (see (15))

$$N_{jt+1} = \left(\frac{1 + \omega_j}{1 + \rho_{jt}} \right) N_{jt}. \quad (A9)$$

1.4 By using (A3) to obtain the labor demanded by each firm producing Y_{jNt} , the intertemporal problem of the representative firm producing nontradables can be solved by maximizing

$$\sum_{v=0}^{\infty} \left[(1 - \gamma_j) \left(\frac{\gamma_j^{\gamma_j} P_{jNt+v} A_{jNt+v} K_{jNt+v}^{1-\gamma_j}}{W_{jt+v}^{\gamma_j}} \right)^{\frac{1}{1-\gamma_j}} - P_{jTt+v} I_{jNt+v} - \lambda_{jNt+v} [K_{jNt+v+1} - I_{jNt+v} - (1 - \delta_j) K_{jNt+v}] \right] \prod_{s=1}^v (1 + i_{jt+s})$$

with respect to I_{jNt} , K_{jNt+1} and the Lagrange multiplier λ_{jNt} , and then by eliminating λ_{jNt} , thus obtaining

$$\frac{(1 - \gamma_j)}{(1 + i_{jt+1})} \left(\frac{\gamma_j^{\gamma_j} P_{jNt+1} A_{jNt+1}}{W_{jt+1}^{\gamma_j}} \right)^{\frac{1}{1-\gamma_j}} + \frac{(1 - \delta_j) P_{jTt+1}}{(1 + i_{jt+1})} = P_{jTt}, \quad (A10)$$

$$K_{jNt+1} = I_{jNt} + (1 - \delta_j)K_{jNt}. \quad (A11)$$

An optimal path must also satisfy the transversality condition

$$\lim_{v \rightarrow \infty} \frac{P_{jTt+v} K_{jNt+v}}{\prod_{s=1}^v (1 + i_{jt+s})} = 0. \quad (A12)$$

1.5 Similarly, one can solve the intertemporal problem of the representative firm producing tradables, thus obtaining

$$\frac{(1 - \alpha_j)}{(1 + i_{jt+1})} \left(\frac{\alpha_j^{\alpha_j} P_{jTt+1} A_{jTt+1}}{W_{jt+1}^{\alpha_j}} \right)^{\frac{1}{1 - \alpha_j}} + \frac{(1 - \delta_j) P_{jTt+1}}{(1 + i_{jt+1})} = P_{jTt}, \quad (A13)$$

$$K_{jTt+1} = I_{jTt} + (1 - \delta_j)K_{jTt}, \quad (A14)$$

$$\lim_{v \rightarrow \infty} \frac{P_{jTt+v} K_{jTt+v}}{\prod_{s=1}^v (1 + i_{jt+s})} = 0. \quad (A15)$$

1.6 By using (14) to obtain the labor supplied by each household, the intertemporal problem of the representative household can be solved by maximizing

$$\sum_{v=0}^{\infty} \theta_j^v \left\{ \ln \left(\frac{C_{jNt+v}^{\eta_j}}{C_{jTt+v}^{\eta_j-1}} \right) + \chi_j \ln \left(\frac{M_{jt+v}}{P_{jt+v}} \right) + v(G_{jt+v}) + \lambda_{jHt+v} [(1 + i_{jt+v})B_{jHt+v} + \pi_{jNt+v} + \right. \\ \left. + \pi_{jTt+v} + E_{jt+v}(1 + i_{it+v})F_{jHt+v} + L_{jt+v}W_{jt+v} + (H_j - L_{jt+v})S_{jt+v} + M_{jt+v-1} - \right. \\ \left. - T_{jt+v} - B_{jHt+v+1} - E_{jt+v}F_{jHt+v+1} - M_{jt+v} - P_{jNt+v}C_{jNt+v} - P_{jTt+v}C_{jTt+v}] \right\}$$

with respect to C_{jNt} , C_{jTt} , M_{jt} , B_{jHt+1} , F_{jHt+1} and the Lagrange multiplier λ_{jHt} , and then by eliminating λ_{jHt} , thus obtaining

$$\eta_j P_{jTt} C_{jTt} = (1 - \eta_j) P_{jNt} C_{jNt}, \quad (A16)$$

$$\chi_j [(1 - \eta_j) M_{jt}]^{-1} = (P_{jTt} C_{jTt})^{-1} - \theta_j (P_{jTt+1} C_{jTt+1})^{-1}, \quad (A17)$$

$$P_{jTt+1} C_{jTt+1} = \theta_j P_{jTt} C_{jTt} (1 + i_{jt+1}), \quad (A18)$$

$$E_{jt} P_{jTt+1} C_{jTt+1} = \theta_j P_{jTt} C_{jTt} E_{jt+1} (1 + i_{it+1}), \quad i \neq j, \quad t \geq t^*, \quad (A19)$$

$$E_{jt} (F_{jHt+1} + F_{jGt+1}) - (1 + i_{jt}) (B_{jHt} + B_{jGt}) - E_{jt} (1 + i_{it}) (F_{jHt} + F_{jGt}) + B_{jHt+1} + B_{jGt+1} = \\ = P_{jTt} [Y_{jTt} - K_{jTt+1} - K_{jNt+1} - C_{jTt} - G_{jTt} + (K_{jTt} + K_{jNt})(1 - \delta_j)], \quad i \neq j. \quad (A20)$$

Notice that (A20) is obtained by using (19) (the government's budget constraint) for substituting T_{jt} in the household's budget constraint, and by using (2), (3), (6), (7), (18), (21) and (22).

The household's optimal path must also satisfy the transversality conditions

$$\lim_{v \rightarrow \infty} \frac{\theta_j^v (1 - \eta_j) B_{jHt+v+1}}{C_{jTt+v} P_{jTt+v}} = 0, \quad (A21)$$

$$\lim_{v \rightarrow \infty} \frac{\theta_j^v (1 - \eta_j) E_{jt+v+1} F_{jHt+v+1}}{C_{jTt+v} P_{jTt+v}} = 0. \quad (A22)$$

1.7 By considering (18) and that the government produces efficiently ($G_{jNt} = \zeta_j G_{jTt}$), one can obtain

$$G_{jTt} = \frac{g_{jt}(P_{jNt} Y_{jNt} + P_{jTt} Y_{jTt})}{P_{jNt} \zeta_j + P_{jTt}} = \frac{g_{jt} \left(\frac{P_{jNt} Y_{jNt}}{P_{jTt}} + Y_{jTt} \right)}{\left(\frac{P_{jNt} \zeta_j}{P_{jTt}} + 1 \right)}. \quad (A23)$$

By using (A5) and the production functions (1) and (5), one can rewrite (A23) as

$$\frac{G_{jTt}}{K_{jTt}} = G \left(\frac{K_{jNt}}{K_{jTt}}, L_{jNt}, L_{jTt}, g_{jt} \right) = \frac{L_{jTt}^{\alpha_j} g_{jt} \left[\frac{\alpha_j L_{jNt}}{\gamma_j L_{jTt}} + 1 \right]}{\frac{\zeta_j \alpha_j K_{jTt} L_{jNt}^{1-\gamma_j}}{\gamma_j K_{jNt} L_{jTt}^{1-\alpha_j}} + 1}. \quad (A24)$$

1.8 By using (A24), $G_{jNt} = \zeta_j G_{jTt}$, the equilibrium condition (22) and the production function (1), one has

$$C_{jNt} = K_{jNt} L_{jNt}^{\gamma_j} - \frac{K_{jTt} L_{jTt}^{\alpha_j} \zeta_j g_{jt} \left[\frac{\alpha_j L_{jNt}}{\gamma_j L_{jTt}} + 1 \right]}{\frac{\zeta_j \alpha_j K_{jTt} L_{jNt}^{1-\gamma_j}}{\gamma_j K_{jNt} L_{jTt}^{1-\alpha_j}} + 1}. \quad (A25)$$

Moreover, one can use (A5) and (A16) to obtain

$$C_{jTt} = \frac{(1 - \eta_j) \alpha_j K_{jTt} L_{jNt}^{1-\gamma_j} C_{jNt}}{\eta_j \gamma_j K_{jNt} L_{jTt}^{1-\alpha_j}}. \quad (A26)$$

Finally, one can use (A25) to rewrite (A26) as

$$\frac{C_{jTt}}{K_{jTt}} = C \left(\frac{K_{jNt}}{K_{jTt}}, L_{jNt}, L_{jTt}, g_{jt} \right) = \frac{(1 - \eta_j) \alpha_j L_{jNt}}{\eta_j \gamma_j L_{jTt}^{1-\alpha_j}} \left[1 - \frac{\frac{K_{jTt} L_{jTt}^{\alpha_j} \zeta_j g_{jt} \left(\frac{\alpha_j L_{jNt}}{\gamma_j L_{jTt}} + 1 \right)}{K_{jNt} L_{jNt}^{\gamma_j} \left(\frac{\alpha_j L_{jNt}}{\gamma_j L_{jTt}} + 1 \right)}}{\left(\frac{\zeta_j \alpha_j K_{jTt} L_{jNt}^{1-\gamma_j}}{\gamma_j K_{jNt} L_{jTt}^{1-\alpha_j}} + 1 \right)} \right]. \quad (A27)$$

1.9 By using (A4) and the fact that $A_{jTt} = K_{jTt}^{\alpha_j}$, one can rewrite (A13) as

$$\frac{(1 - \alpha_j) L_{jTt+1}^{\alpha_j} P_{jTt+1}}{(1 + i_{jt+1})} + \frac{(1 - \delta_j) P_{jTt+1}}{(1 + i_{jt+1})} = P_{jTt}. \quad (A28)$$

By using (A18), one can rearrange (A28), thus obtaining

$$\theta_j[(1-\alpha_j)L_{jTt+1}^{\alpha_j} + (1-\delta_j)]\frac{C_{jTt}}{C_{jTt+1}} = 1. \quad (A29)$$

Finally, one can use (A27) to rewrite (A29) as

$$\rho_{jt} = \frac{\theta_j[(1-\alpha_j)L_{jTt+1}^{\alpha_j} + (1-\delta_j)]C\left(\frac{K_{jNt}}{K_{jTt}}, L_{jNt}, L_{jTt}, g_{jt}\right)}{C\left(\frac{K_{jNt+1}}{K_{jTt+1}}, L_{jNt+1}, L_{jTt+1}, g_{jt+1}\right)} - 1. \quad (A30)$$

1.10 By using (A3) and the fact that $A_{jNt} = K_{jNt}^{\gamma_j}$, one can rewrite (A10) as

$$\frac{(1-\gamma_j)L_{jNt+1}^{\gamma_j}P_{jNt+1}}{(1+i_{jt+1})} + \frac{(1-\delta_j)P_{jTt+1}}{(1+i_{jt+1})} = P_{jTt}. \quad (A31)$$

Moreover, one can use (A5), (A28) and (A31) to obtain

$$\frac{K_{jNt}}{K_{jTt}} = \begin{cases} K(L_{jNt}, L_{jTt}) = \frac{\alpha_j(1-\gamma_j)L_{jNt}}{\gamma_j(1-\alpha_j)L_{jTt}} & \text{if } t > 0 \\ \frac{K_{jN0}}{K_{jT0}} & \text{otherwise.} \end{cases} \quad (A32)$$

Finally, one can consider (A8) and (A32) to check that for $t > 0$ one has $L_{jNt} = H_j - L_{jTt}$

$$\left(\text{with } L_{jTt} \leq \left[\left(\frac{1-\gamma_j}{1-\alpha_j} \right)^{\eta_j} \frac{(H_j - L_{jTt})^{\eta_j} \alpha_j D_j}{N_{jt}} \right]^{\frac{1}{1-\alpha_j(1-\eta_j)}} \right) \text{ along an equilibrium path characterized by full}$$

employment, i.e., an equilibrium path where all labor is employed in the market sectors of the economy,

$$\text{and } L_{jNt} = \left[\left(\frac{1-\alpha_j}{1-\gamma_j} \right)^{\eta_j} \frac{N_{jt} L_{jTt}^{1-\alpha_j(1-\eta_j)}}{\alpha_j D_j} \right]^{\frac{1}{\gamma_j \eta_j}} < H_j - L_{jTt} \text{ along an equilibrium path where some labor is not}$$

employed in the market sectors of the economy. Hence, one can conclude that

$$L_{jNt} = m(N_{jt}, L_{jTt}) = \begin{cases} H_j - L_{jTt} & \text{if country } j \text{ moves along an equilibrium path} \\ & \text{where all labor is employed in the market sectors} \\ \left[\left(\frac{1-\alpha_j}{1-\gamma_j} \right)^{\eta_j} \frac{N_{jt} L_{jTt}^{1-\alpha_j(1-\eta_j)}}{\alpha_j D_j} \right]^{\frac{1}{\gamma_j \eta_j}} & \text{if country } j \text{ moves} \\ & \text{along an equilibrium path where some labor is not} \\ & \text{employed in the market sectors, } t > 0. \end{cases} \quad (A33)$$

1.11 By rewriting (A17) as

$$\chi_j(1-\eta_j)^{-1} = x_{jt} - x_{jt+1} \theta_j(1+\bar{\mu}_j)^{-1}, \quad x_{jt} \equiv M_{jt}(P_{jTt}C_{jTt})^{-1}, \quad (A34)$$

Since $\theta(1 + \bar{\mu}_j)^{-1} < 1$, equation (A34) is such that if $x_{j0} > x_j$ then $x_{jt} \rightarrow \infty$ as $t \rightarrow \infty$, if $x_{j0} < x_j$ then $x_{jt} \rightarrow -\infty$ as $t \rightarrow \infty$, if $x_{j0} = x_j$ then $x_{jt} = x_j$ for all t , where $x_j = \frac{\chi_j(1 + \bar{\mu}_j)}{(1 - \eta_j)(1 + \bar{\mu}_j - \theta_j)}$. Therefore, the only value of x_{jt} that is consistent with the optimality and boundary conditions is $x_{jt} = x_j$ for all t . This implies that along an equilibrium path one has

$$P_{jTt} = \frac{M_{jt}}{C_{jTt}x_j} = \frac{(1 - \eta_j)(1 + \bar{\mu}_j - \theta_j)(1 + \bar{\mu}_j)^t M_{j-1}}{C_{jTt}\chi_j}. \quad (A35)$$

Considering (A35), one can use the one-price law (24) to obtain

$$E_{jt} = \frac{(1 + \bar{\mu}_j - \theta_j)(1 - \eta_j)\chi_i(1 + \bar{\mu}_j)^t M_{j-1}C_{iTt}}{(1 + \bar{\mu}_i - \theta_i)(1 - \eta_i)\chi_j(1 + \bar{\mu}_i)^t M_{i-1}C_{jTt}}, i \neq j. \quad (A36)$$

1.12 Since the rate of money growth is fixed in both countries, the equilibrium level of the nominal interest rate in country j is constant:

$$i_{jt} = \begin{cases} \frac{1 + \bar{\mu}_j}{\theta_j} - 1 & \text{if } t > 0 \\ i_{j0} & \text{otherwise, } i_{j0} \text{ given.} \end{cases} \quad (A37)$$

1.13 By using (A35), the equilibrium level of the trade account of country j (denominated in domestic currency) can be written as

$$TA_{jt} = \frac{\left[L_{jTt}^{\alpha_j} - \frac{C_{jTt}}{K_{jTt}} - \frac{G_{jTt}}{K_{jTt}} - \left(1 + \frac{K_{jNt+1}}{K_{jTt+1}} \right) (1 + \rho_{jt}) + (1 - \delta_j) \left(1 + \frac{K_{jNt}}{K_{jTt}} \right) \right]}{\frac{C_{jTt}}{K_{jTt}} \chi_j [(1 - \eta_j)(1 + \bar{\mu}_j - \theta_j) M_{j-1} (1 + \bar{\mu}_j)^t]^{-1}}. \quad (A38)$$

2 Derivation of equation (35)

Considering (31) and (A18), one can check that

$$\frac{E_{jt}}{E_{j,t-1}} = \frac{1 + i_{jt}}{1 + i_{jt-1}} = \frac{P_{jTt}P_{iTt-1}\theta_i C_{jTt}C_{iTt-1}}{P_{iTt}P_{jTt-1}\theta_j C_{jTt-1}C_{iTt}}, i \neq j, t \geq t^*. \quad (A39)$$

Considering (24), one has

$$\frac{E_{jt}}{E_{j,t-1}} = \frac{P_{jTt}P_{iTt-1}}{P_{iTt}P_{jTt-1}}, i \neq j. \quad (A40)$$

Thus, (A39) and (A40)—together—imply that in phase 2 one has $\frac{\theta_i C_{jTt}C_{iTt-1}}{\theta_j C_{jTt-1}C_{iTt}} = 1$, $i \neq j$, $t \geq t^*$, which in

its turn entails (35) (see equation (A29)).

3 Derivation of the difference equations governing the equilibrium path of the economy

3.1 One can use (A8) (with $L_{usNt} = H_{us} - L_{usTt}$), (A24), (A27), (A30) and (A32) to write (A1) as

$$\begin{aligned} \frac{TA_{ust}}{P_{usTt}K_{usTt}} + \frac{TA_{cht}}{P_{chtTt}K_{chtTt}} &= 0, \text{ where} \\ \frac{-TA_{ust}}{P_{usTt}K_{usTt}} &= \frac{[1 + K(H_{us} - L_{usTt+1}, L_{usTt+1})]\theta_{us}C(K(H_{us} - L_{usTt}, L_{usTt}), H_{us} - L_{usTt}, L_{usTt}, g_{ust})}{[(1 - \alpha_{us})L_{usTt+1}^{\alpha_{us}} + 1 - \delta_{us}]^{-1}C(K(H_{us} - L_{usTt+1}, L_{usTt+1}), H_{us} - L_{usTt+1}, L_{usTt+1}, g_{ust+1})} - \\ &\quad - (1 - \delta_{us})[1 + K(H_{us} - L_{usTt}, L_{usTt})] + G(K(H_{us} - L_{usTt}, L_{usTt}), H_{us} - L_{usTt}, L_{usTt}, g_{ust}) + \\ &\quad + C(K(H_{us} - L_{usTt}, L_{usTt}), H_{us} - L_{usTt}, L_{usTt}, g_{ust}) - L_{usTt}^{\alpha_{us}}, t > 0, \quad (A41) \end{aligned}$$

and

$$\begin{aligned} -\frac{TA_{cht}}{P_{chtTt}K_{chtTt}} &= \frac{[1 + K(L_{chtNt+1}, L_{chtTt+1})]\theta_{ch}C(K(L_{chtNt}, L_{chtTt}), L_{chtNt}, L_{chtTt}, g_{cht})}{[(1 - \alpha_{ch})L_{chtTt+1}^{\alpha_{ch}} + 1 - \delta_{ch}]^{-1}C(K(L_{chtNt+1}, L_{chtTt+1}), L_{chtNt+1}, L_{chtTt+1}, g_{cht+1})} - \\ &\quad - (1 - \delta_{ch})[1 + K(L_{chtNt}, L_{chtTt})] - L_{chtTt}^{\alpha_{ch}} + G(K(L_{chtNt}, L_{chtTt}), L_{chtNt}, L_{chtTt}, g_{cht}) + \\ &\quad + C(K(L_{chtNt}, L_{chtTt}), L_{chtNt}, L_{chtTt}, g_{cht}), t > 0. \quad (A42) \end{aligned}$$

Similarly, one can use (A8) (with $L_{usNt} = H_{us} - L_{usTt}$), (A24), (A27), (A30) and (A32) to write (A2) as

$$\begin{aligned} Z_{t+1} - \frac{\frac{Z_t\theta_{us}C(K(H_{us} - L_{usTt}, L_{usTt}), H_{us} - L_{usTt}, L_{usTt}, g_{ust})}{C(K(H_{us} - L_{usTt+1}, L_{usTt+1}), H_{us} - L_{usTt+1}, L_{usTt+1}, g_{ust+1})}}{[(1 - \alpha_{ch})L_{chtTt+1}^{\alpha_{ch}} + 1 - \delta_{ch}]\theta_{ch}C(K(L_{chtNt}, L_{chtTt}), L_{chtNt}, L_{chtTt}, g_{cht})} \\ \frac{[(1 - \alpha_{us})L_{usTt+1}^{\alpha_{us}} + 1 - \delta_{us}]C(K(L_{chtNt+1}, L_{chtTt+1}), L_{chtNt+1}, L_{chtTt+1}, g_{cht+1})}{(A43)} = 0, t > 0. \end{aligned}$$

Finally, one can use (A24), (A27), (A30) and (A32) to write (A9) as

$$N_{cht+1} - \frac{N_{cht}(1 + \omega_{cht})C(K(L_{chtNt+1}, L_{chtTt+1}), L_{chtNt+1}, L_{chtTt+1}, g_{cht+1})}{\theta_{ch}[(1 - \alpha_{ch})L_{chtTt+1}^{\alpha_{ch}} + 1 - \delta_{ch}]C(K(L_{chtNt}, L_{chtTt}), L_{chtNt}, L_{chtTt}, g_{cht})} = 0, t > 0. \quad (A44)$$

3.2 In phase 1, we assume for simplicity and without loss of generality that $g_{ust} = \hat{g}_{us}$, $g_{cht} = \hat{g}_{ch}$ and $\omega_{cht} = \hat{\omega}_{ch} \forall t$ such that $0 \leq t < t^*$. Hence, the equilibrium path of the economy is governed for $0 < t < t^*$

by a system of difference equations in L_{chTt} , $Z_t \equiv \frac{K_{usTt}}{K_{chtTt}}$ and $N_{cht} \equiv \frac{V_{cht}}{K_{chtTt}}$:

$$\begin{aligned} \Omega(L_{chTt+1}, N_{cht+1}, L_{chTt}, N_{cht}, Z_t, \hat{g}_{us}, \hat{g}_{ch}, Q) &= Z_t n(L_{chTt+1}, N_{cht+1}, L_{chTt}, N_{cht}, \hat{g}_{us}, \hat{g}_{ch}, Q) + \\ &\quad + y(L_{chTt+1}, N_{cht+1}, L_{chTt}, N_{cht}, \hat{g}_{ch}) = 0, \quad Q > \underline{Q}_t, \quad 0 < t < t^*, \quad (A45) \end{aligned}$$

$$\Phi(L_{chTt+1}, Z_{t+1}, N_{cht+1}, L_{chTt}, Z_t, N_{cht}, \hat{g}_{us}, \hat{g}_{ch}, Q) = 0, \quad Q > \underline{Q}_t, \quad 0 < t < t^*, \quad (A46)$$

$$\Theta(L_{chTt+1}, N_{cht+1}, L_{chTt}, N_{cht}, \hat{g}_{ch}, \hat{\omega}_{ch}) = 0, \quad 0 < t < t^*,^{45} \quad (A47)$$

where $n(L_{chTt+1}, N_{cht+1}, L_{chTt}, N_{cht}, \hat{g}_{us}, \hat{g}_{ch}, Q) = -\frac{TA_{ust}}{P_{usTt}K_{usTt}}$ is obtained by setting

$L_{usTt} = f(L_{chTt}, m(N_{cht}, L_{chTt}), \hat{g}_{us}, \hat{g}_{ch}, Q)$ (see (38) and (A33)), $g_{ust} = \hat{g}_{us}$ and $g_{cht} = \hat{g}_{ch} \forall t$ such

⁴⁵ At time 0, the dynamics of the economy depends also on the initial endowments of capital K_{chT0} , K_{usT0} , K_{chN0} and K_{usN0} (see further).

that $0 < t < t^*$ in (A41); $y(L_{cht+1}, N_{cht+1}, L_{cht}, N_{cht}, \hat{g}_{ch}) = -\frac{TA_{cht}}{P_{cht}K_{cht}}$ is obtained by setting

$L_{chNt} = m(N_{cht}, L_{cht})$ and $g_{cht} = \hat{g}_{ch} \forall t$ such that $0 < t < t^*$ in (A42); equation (A46) is obtained by setting $L_{chNt} = m(N_{cht}, L_{cht})$, $L_{usTt} = f(L_{cht}, m(N_{cht}, L_{cht}), \hat{g}_{ch}, \hat{g}_{us}, Q)$, $g_{ust} = \hat{g}_{us}$ and $g_{cht} = \hat{g}_{ch} \forall t$ such that $0 < t < t^*$ in (A43); equation (A47) is obtained by setting $L_{chNt} = m(N_{cht}, L_{cht})$, $g_{cht} = \hat{g}_{ch}$ and $\omega_{cht} = \hat{\omega}_{ch} \forall t$ such that $0 < t < t^*$ in (A44).

3.3 Notice that at $t=0$ the dynamics of the economy is governed by

$$\begin{aligned} Z_0 & \left\{ G\left(\frac{K_{usN0}}{K_{usT0}}, H_{us} - L_{usT0}, L_{usT0}, \hat{g}_{us}\right) + C\left(\frac{K_{usN0}}{K_{usT0}}, H_{us} - L_{usT0}, L_{usT0}, \hat{g}_{us}\right) - L_{usT0}^{\alpha_{us}} + \right. \\ & \left. + \frac{[1 + K(H_{us} - L_{usT1}, L_{usT1})]\theta_{us} C\left(\frac{K_{usN0}}{K_{usT0}}, H_{us} - L_{usT0}, L_{usT0}, \hat{g}_{us}\right)}{\left[(1 - \alpha_{us})L_{usT1}^{\alpha_{us}} + 1 - \delta_{us}\right]^{-1}} - (1 - \delta_{us})\left(1 + \frac{K_{usN0}}{K_{chT0}}\right) \right\} + \\ & + G\left(\frac{K_{chN0}}{K_{chT0}}, L_{chN0}, L_{chT0}, \hat{g}_{ch}\right) - L_{chT0}^{\alpha_{ch}} + C\left(\frac{K_{chN0}}{K_{chT0}}, L_{chN0}, L_{chT0}, \hat{g}_{ch}\right) - (1 - \delta_{ch})\left(1 + \frac{K_{chN0}}{K_{chT0}}\right) + \\ & + \frac{[1 + K(L_{chN1}, L_{chT1})]\theta_{ch} C\left(\frac{K_{chN0}}{K_{chT0}}, L_{chN0}, L_{chT0}, \hat{g}_{ch}\right)}{\left[(1 - \alpha_{us})L_{chT1}^{\alpha_{ch}} + 1 - \delta_{ch}\right]^{-1}} C(K(L_{chN1}, L_{chT1}), L_{chN1}, L_{chT1}, \hat{g}_{ch}) = 0, \quad (A48) \end{aligned}$$

$$\begin{aligned} Z_0 \theta_{us} C\left(\frac{K_{usN0}}{K_{usT0}}, H_{us} - L_{usT0}, L_{usT0}, \hat{g}_{us}\right) \\ Z_1 - \frac{C(K(H_{us} - L_{usT1}, L_{usT1}), H_{us} - L_{usT1}, L_{usT1}, \hat{g}_{us})}{[(1 - \alpha_{ch})L_{chT1}^{\alpha_{ch}} + 1 - \delta_{ch}]\theta_{ch} C\left(\frac{K_{chN0}}{K_{chT0}}, L_{chN0}, L_{chT0}, \hat{g}_{ch}\right)} = 0 \quad (A49) \\ \frac{C(K(L_{chN1}, L_{chT1}), L_{chN1}, L_{chT1}, \hat{g}_{ch})}{[(1 - \alpha_{us})L_{usT1}^{\alpha_{us}} + 1 - \delta_{us}]C(K(L_{chN1}, L_{chT1}), L_{chN1}, L_{chT1}, \hat{g}_{ch})} \end{aligned}$$

and

$$N_{ch1} - \frac{N_{ch0}(1 + \hat{\omega}_{ch})C(K(L_{chN1}, L_{chT1}), L_{chN1}, L_{chT1}, \hat{g}_{ch})}{\theta_{ch}[(1 - \alpha_{ch})L_{chT1}^{\alpha_{ch}} + 1 - \delta_{ch}]C\left(\frac{K_{chN0}}{K_{chT0}}, L_{chN0}, L_{chT0}, \hat{g}_{ch}\right)} = 0, \quad (A50)$$

where

$$L_{chN0} = L\left(\frac{K_{chN0}}{K_{chT0}}, N_{ch0}, L_{chT0}\right) < H_{ch} - L_{chT0} \quad \text{and}$$

$L_{usT0} = f(L_{chT0}, L\left(\frac{K_{chN0}}{K_{chT0}}, N_{ch0}, L_{chT0}\right), \hat{g}_{ch}, \hat{g}_{us}, Q)$, where N_{ch0} can be controlled by the Chinese

authorities (which decide about the government transfer s_{ch0}), and where Z_0 , $\frac{K_{chN0}}{K_{chT0}}$ and $\frac{K_{usN0}}{K_{usT0}}$ are

exogenously given.

3.4 In phase 2, we assume for simplicity and without loss of generality that $g_{ust} = \bar{g}_{us}$, $g_{cht} = \bar{g}_{ch}$ and $\omega_{cht} = \bar{\omega}_{ch} \forall t \geq t^*$. Hence, the equilibrium path of the economy is governed for $t \geq t^*$ by a system of difference equations in L_{chTt} , Z_t and N_{cht} :

$$\Psi(L_{chTt+1}, N_{cht+1}, L_{chTt}, Z_t, N_{cht}, \bar{g}_{us}, \bar{g}_{ch}) = Z_t b(L_{chTt+1}, L_{chTt}, \bar{g}_{us}) + y(L_{chTt+1}, N_{cht+1}, L_{chTt}, N_{cht}, \bar{g}_{ch}) = 0, t \geq t^*, \quad (A51)$$

$$\Lambda(L_{chTt+1}, Z_{t+1}, N_{cht+1}, L_{chTt}, Z_t, N_{cht}, \bar{g}_{us}, \bar{g}_{ch}) = 0, t \geq t^*, \quad (A52)$$

$$\Theta(L_{chTt+1}, N_{cht+1}, L_{chTt}, N_{cht}, \bar{g}_{ch}, \bar{\omega}_{ch}) = 0, t \geq t^*, \quad (A53)$$

where $b(L_{chTt+1}, L_{chTt}, \bar{g}_{us}) = -\frac{TA_{ust}}{P_{usTt} K_{usTt}}$ is obtained by setting $L_{usTt} = l(L_{chTt})$ (see equation (35))

and $g_{ust+1} = g_{ust} = \bar{g}_{us}$ in (A41); and equation (A52) is obtained by setting $L_{usTt} = l(L_{chTt})$, $L_{jNt} = m(N_{jt}, L_{jTt})$, $g_{ust+1} = g_{ust} = \bar{g}_{us}$ and $g_{cht+1} = g_{cht} = \bar{g}_{ch}$ in (A43).

The constraint imposed on US policies by the Chinese willingness to finance the US external deficit amounts to

$$b(L_{chTt+1}, L_{chTt}, \bar{g}_{us}) = \frac{-TA_{ust}}{P_{usTt} K_{usTt}} \leq \frac{\xi[P_{usNt} Y_{usNt} + P_{usTt} Y_{usTt}]}{P_{usTt} K_{usTt}}. \text{ By using (1), (5), (A5), (A32),}$$

$L_{usNt} = H_{us} - L_{usTt}$ and $L_{usTt} = l(L_{chTt})$, this constraint can be written as

$$b(L_{chTt+1}, L_{chTt}, \bar{g}_{us}) \leq \xi[l(L_{chTt})]^{\alpha_{us}} \left\{ \frac{\alpha_{us}[H_{us} - l(L_{chTt})]}{\gamma_{us} l(L_{chTt})} + 1 \right\}, t \geq t^*. \quad (A54)$$

4 Asymptotic behavior of the economy

We study the asymptotic behavior of the economy under the hypothesis that the Chinese households are less impatient than their US counterparts ($\theta_{ch} > \theta_{us}$).

4.1 Suppose that $t^* < \infty$. In this case, an equilibrium path of the economy must satisfy (A51)-(A54) for $t \geq t^*$ (phase 2) and be such that $L_{chTt} \rightarrow L_{chT}$, $Z_t \rightarrow Z$ and $N_{cht} \rightarrow N_{ch}$ as $t \rightarrow \infty$, where $0 \leq L_{chT} \leq H_{ch}$, $Z=0$ and $0 \leq N_{ch} < \infty$.

In their turn, $L_{jTt} \rightarrow L_{jT}$ and $N_{jt} \rightarrow N_j$ as $t \rightarrow \infty$ imply that country j's rate of real GDP growth approaches

$$\rho_j = \theta_j[(1 - \alpha_j)L_{jT}^{\alpha_j} + 1 - \delta_j] - 1, \text{ where } \rho_j = \lim_{t \rightarrow \infty} \rho_{jt}, \text{ thus entailing } \frac{\partial \rho_j}{\partial L_{jT}} > 0. \text{ This can be shown by}$$

using (1), (5), (11), (A5), (A30), (A32) and (A33) in order to verify that the country j's rate of real GDP growth is given by

$$\rho_{GDP_{jt}} = (1 + \rho_{jt}) \frac{\frac{[m(N_{jt+1}, L_{jTt+1})]^{\gamma_j \eta_j} L_{jTt+1}^{\alpha_j(1-\eta_j)-1}}{[m(N_{jt}, L_{jTt})]^{\gamma_j \eta_j} L_{jTt}^{\alpha_j(1-\eta_j)-1}}}{\left[\frac{\gamma_j L_{jTt} + \alpha_j m(N_{jt}, L_{jTt})}{\gamma_j L_{jTt+1} + \alpha_j m(N_{jt+1}, L_{jTt+1})} \right]} - 1, t \geq t^*, \quad (A55)$$

$$\text{where } \rho_{\text{GDP}_{jt}} \equiv \frac{\frac{P_{jTt+1}Y_{jTt+1} + P_{jNt+1}Y_{jNt+1}}{P_{jt+1}} - \left(\frac{P_{jTt}Y_{jTt} + P_{jNt}Y_{jNt}}{P_{jt}} \right)}{\frac{P_{jTt}Y_{jTt} + P_{jNt}Y_{jNt}}{P_{jt}}}.$$

By inspecting (A55), one can easily check that $L_{jTt} \rightarrow L_{jT}$ and $N_{jt} \rightarrow N_j$ as $t \rightarrow \infty$ imply that

$\lim_{t \rightarrow \infty} \rho_{\text{GDP}_{jt}} = \lim_{t \rightarrow \infty} \rho_{jt}$. Moreover, by considering (A30), (A32) and (A33), one can also check that

$L_{jTt} \rightarrow L_{jT}$ and $N_{jt} \rightarrow N_j$ as $t \rightarrow \infty$, where $0 \leq L_{jT} \leq H_j$ and $0 \leq N_j < \infty$, imply that

$\lim_{t \rightarrow \infty} \rho_{jt} = \rho_j = \theta_j[(1 - \alpha_j)L_{jT}^{\alpha_j} + 1 - \delta_j] - 1$. Thus, $L_{jTt} \rightarrow L_{jT}$ and $N_{jt} \rightarrow N_j$ as $t \rightarrow \infty$ entail

$\lim_{t \rightarrow \infty} \rho_{\text{GDP}_{jt}} = \theta_j[(1 - \alpha_j)L_{jT}^{\alpha_j} + 1 - \delta_j] - 1$. Finally, by considering (A9) and (A33), one can conclude that

$\rho_j = \theta_j[(1 - \alpha_j)L_{jT}^{\alpha_j} + 1 - \delta_j] - 1 \geq \bar{\omega}_j$ is a necessary condition for the existence of an equilibrium path.

Given (35), it is trivial to see that i) $L_{\text{ch}Tt} \rightarrow L_{\text{ch}T}$ entails $L_{\text{us}Tt} \rightarrow L_{\text{us}T}$, and that ii)

$(1 - \alpha_{\text{us}})L_{\text{us}T}^{\alpha_{\text{us}}} + 1 - \delta_{\text{us}} = (1 - \alpha_{\text{ch}})L_{\text{ch}T}^{\alpha_{\text{ch}}} + 1 - \delta_{\text{ch}}$, which entails $\rho_{\text{ch}} > \rho_{\text{us}}$ since $\theta_{\text{ch}} > \theta_{\text{us}}$. In its turn,

$\rho_{\text{ch}} > \rho_{\text{us}}$ implies that $Z_t \rightarrow 0$ as $t \rightarrow \infty$.

As $t \rightarrow \infty$ and $Z_t \rightarrow 0$, equation (A51) becomes

$$y(L_{\text{ch}Tt+1} = L_{\text{ch}T}, N_{\text{cht}+1} = N_{\text{ch}T}, L_{\text{ch}Tt} = L_{\text{ch}T}, N_{\text{cht}} = N_{\text{ch}T}, \bar{g}_{\text{ch}}) = 0 \quad (\text{A56})$$

where

$$y(\cdot) = \frac{\left[1 + \frac{(1 - \gamma_{\text{ch}})\alpha_{\text{ch}}m(N_{\text{ch}}, L_{\text{ch}T})}{(1 - \alpha_{\text{ch}})\gamma_{\text{ch}}L_{\text{ch}T}} \right]}{[\theta_{\text{ch}}(1 - \alpha_{\text{ch}})L_{\text{ch}T}^{\alpha_{\text{ch}}} - (1 - \delta_{\text{ch}})(1 - \theta_{\text{ch}})]^{-1}} + \frac{(1 - \eta_{\text{ch}})\alpha_{\text{ch}}m(N_{\text{ch}}, L_{\text{ch}T})}{\eta_{\text{ch}}\gamma_{\text{ch}}L_{\text{ch}T}^{1-\alpha_{\text{ch}}}} - L_{\text{ch}T}^{\alpha_{\text{ch}}} -$$

$$- \left[\frac{\left[\frac{(1 - \eta_{\text{ch}})(1 - \alpha_{\text{ch}})L_{\text{ch}T}^{\alpha_{\text{ch}}}\zeta_{\text{ch}}}{\eta_{\text{ch}}(1 - \gamma_{\text{ch}})[m(N_{\text{ch}}, L_{\text{ch}T})]^{\gamma_{\text{ch}}}} - 1 \right] \left(\frac{\alpha_{\text{ch}}m(N_{\text{ch}}, L_{\text{ch}T})}{\gamma_{\text{ch}}L_{\text{ch}T}} + 1 \right)}{\left(\frac{(1 - \alpha_{\text{ch}})L_{\text{ch}T}^{\alpha_{\text{ch}}}\zeta_{\text{ch}}}{(1 - \gamma_{\text{ch}})[m(N_{\text{ch}}, L_{\text{ch}T})]^{\gamma_{\text{ch}}}} + 1 \right)} \right] L_{\text{ch}T}^{\alpha_{\text{ch}}}\bar{g}_{\text{ch}}.$$

An asymptotic equilibrium pair $(L_{\text{ch}T}, N_{\text{ch}})$ must i) satisfy (A56) and be such that

ii) $\rho_{\text{ch}} = \theta_{\text{ch}}[(1 - \alpha_{\text{ch}})L_{\text{ch}T}^{\alpha_{\text{ch}}} + 1 - \delta_{\text{ch}}] - 1 \geq \bar{\omega}_{\text{ch}}$

$\rho_{\text{us}} = \theta_{\text{us}} \left\{ (1 - \alpha_{\text{us}})[L_{\text{ch}T}]^{\alpha_{\text{us}}} + 1 - \delta_{\text{us}} \right\} - 1 \geq \bar{\omega}_{\text{us}}$ (see (35) and (A9)), and

$$\text{iii) } L_{\text{chT}} \leq \left[\left(\frac{1 - \gamma_{\text{ch}}}{1 - \alpha_{\text{ch}}} \right)^{\eta_{\text{ch}}} \frac{(H_{\text{ch}} - L_{\text{chT}})^{\eta_{\text{ch}} \gamma_{\text{ch}}} \alpha_{\text{ch}} D_{\text{ch}}}{N_{\text{ch}}} \right]^{\frac{1}{1 - \alpha_{\text{ch}}(1 - \eta_{\text{ch}})}},$$

$$l(L_{\text{chT}}) \leq \left[\left(\frac{1 - \gamma_{\text{us}}}{1 - \alpha_{\text{us}}} \right)^{\eta_{\text{us}}} \frac{[H_{\text{us}} - l(L_{\text{chT}})]^{\eta_{\text{us}} \gamma_{\text{us}}} \alpha_{\text{us}} D_{\text{us}}}{N_{\text{us}}} \right]^{\frac{1}{1 - \alpha_{\text{us}}(1 - \eta_{\text{us}})}} \quad (\text{see (35) and (A33)}).$$

Typically, there exists two asymptotic equilibrium pairs $(L_{\text{chT}}, N_{\text{ch}})$ that satisfy i), ii) and iii):

$(L'_{\text{chT}}, N'_{\text{ch}})$ and $(L''_{\text{chT}}, N''_{\text{ch}})$, where $L'_{\text{chT}} > L''_{\text{chT}}$, $\rho'_{\text{ch}} > \rho''_{\text{ch}} = \bar{\omega}_{\text{ch}}$, $0 = N'_{\text{ch}} < N''_{\text{ch}} < \infty$, $L'_{\text{chN}} = H_{\text{ch}} - L'_{\text{chT}}$ and

$$L''_{\text{chN}} = \left[\left(\frac{1 - \alpha_{\text{ch}}}{1 - \gamma_{\text{ch}}} \right)^{\eta_{\text{ch}}} \frac{N''_{\text{ch}} (L''_{\text{chT}})^{1 - \alpha_{\text{ch}}(1 - \eta_{\text{ch}})}}{\alpha_{\text{ch}} D_{\text{ch}}} \right]^{\frac{1}{\gamma_{\text{ch}} \eta_{\text{ch}}}} < H_{\text{ch}} - L''_{\text{chT}}.$$

Notice that $(L'_{\text{chT}}, Z' = 0, N'_{\text{ch}} = 0)$ is associated with an equilibrium path along which all China's labor is employed in the market sectors of the economy, while $(L''_{\text{chT}}, Z'' = 0, N''_{\text{ch}})$ is associated with an equilibrium path along which some Chinese labor is not employed in the market sectors of the economy.

Finally, in the special case in which there exists a unique pair $(L_{\text{chT}}, N_{\text{ch}})$ satisfying i), ii) and iii), one has $L_{\text{chN}} = H_{\text{ch}} - L_{\text{chT}}$ and $\rho_{\text{ch}} = \bar{\omega}_{\text{ch}}$.

4.2 Suppose now that $t^* \rightarrow \infty$ and $Q > \underline{Q}_t \quad \forall t > 0$ (the Chinese currency is kept undervalued forever). In

this case, we do not have phase 2: an equilibrium path of the economy must satisfy (A45)-(A47) $\forall t > 0$ and be such that $L_{\text{chT}t} \rightarrow L_{\text{chT}}$, $Z_t \rightarrow Z$ and $N_{\text{cht}} \rightarrow N_{\text{ch}}$ as $t \rightarrow \infty$, where $0 \leq L_{\text{chT}} \leq H_{\text{ch}}$, $Z = 0$ and $0 \leq N_{\text{ch}} < \infty$. As in paragraph 4.1, $L_{jt} \rightarrow L_j$ and $N_{jt} \rightarrow N_j$ as $t \rightarrow \infty$ imply that country j 's rate of real GDP growth approaches $\rho_j = \theta_j[(1 - \alpha_j)L_{jt}^{\alpha_j} + 1 - \delta_j] - 1$.

Given (38) with $Q > \underline{Q}_t$ and given (A33), one can verify that i) $L_{\text{chT}t} \rightarrow L_{\text{chT}}$ and $N_{\text{cht}} \rightarrow N_{\text{ch}}$ entail $L_{\text{usT}t} \rightarrow L_{\text{usT}}$, and that

ii) $(1 - \alpha_{\text{us}})L_{\text{usT}}^{\alpha_{\text{us}}} + 1 - \delta_{\text{us}} < (1 - \alpha_{\text{ch}})L_{\text{chT}}^{\alpha_{\text{ch}}} + 1 - \delta_{\text{ch}}$, which entails $\rho_{\text{ch}} > \rho_{\text{us}}$ since $\theta_{\text{ch}} > \theta_{\text{us}}$. In its turn, $\rho_{\text{ch}} > \rho_{\text{us}}$ implies that $Z_t \rightarrow 0$ as $t \rightarrow \infty$.

As $t \rightarrow \infty$ and $Z_t \rightarrow 0$, equation (A45) becomes

$$y(L_{\text{chT}t+1} = L_{\text{chT}}, N_{\text{cht}+1} = N_{\text{ch}}, L_{\text{chT}t} = L_{\text{chT}}, N_{\text{cht}} = N_{\text{ch}}, \hat{g}_{\text{ch}}) = 0. \quad (\text{A57})$$

An asymptotic equilibrium pair $(L_{\text{chT}}, N_{\text{ch}})$ must i) satisfy (A57) and be such that

ii) $\rho_{\text{ch}} = \theta_{\text{ch}}[(1 - \alpha_{\text{ch}})L_{\text{chT}}^{\alpha_{\text{ch}}} + 1 - \delta_{\text{ch}}] - 1 \geq \hat{\omega}_{\text{ch}}$

$\rho_{\text{us}} = \theta_{\text{us}} \{ [1 - \alpha_{\text{us}}][f(L_{\text{chT}}, m(N_{\text{ch}}, L_{\text{chT}}), \hat{g}_{\text{ch}}, \hat{g}_{\text{us}}, Q)]^{\alpha_{\text{us}}} + 1 - \delta_{\text{us}} \} - 1 \geq \hat{\omega}_{\text{us}}$ (see (38) and (A9)), and

$$\text{iii) } L_{\text{chT}} \leq \left[\left(\frac{1 - \gamma_{\text{ch}}}{1 - \alpha_{\text{ch}}} \right)^{\eta_{\text{ch}}} \frac{(H_{\text{ch}} - L_{\text{chT}})^{\eta_{\text{ch}} \gamma_{\text{ch}}} \alpha_{\text{ch}} D_{\text{ch}}}{N_{\text{ch}}} \right]^{\frac{1}{1 - \alpha_{\text{ch}}(1 - \eta_{\text{ch}})}},$$

$$\begin{aligned} & f(L_{\text{chT}}, m(N_{\text{ch}}, L_{\text{chT}}), \hat{g}_{\text{ch}}, \hat{g}_{\text{us}}, Q) \leq \\ & \leq \left[\left(\frac{1 - \gamma_{\text{us}}}{1 - \alpha_{\text{us}}} \right)^{\eta_{\text{us}}} \frac{[H_{\text{us}} - f(L_{\text{chT}}, m(N_{\text{ch}}, L_{\text{chT}}), \hat{g}_{\text{ch}}, \hat{g}_{\text{us}}, Q)]^{\eta_{\text{us}} \gamma_{\text{us}}} \alpha_{\text{us}} D_{\text{us}}}{N_{\text{us}}} \right]^{\frac{1}{1 - \alpha_{\text{us}}(1 - \eta_{\text{us}})}} \end{aligned}$$

(see (38) and (A33)).

Typically, there exists two asymptotic equilibrium pairs $(L_{\text{chT}}, N_{\text{ch}})$ that satisfy i), ii) and iii):

$(L'_{\text{chT}}, N'_{\text{ch}})$ and $(L''_{\text{chT}}, N''_{\text{ch}})$, where $L'_{\text{chT}} > L''_{\text{chT}}$, $\rho'_{\text{ch}} > \rho''_{\text{ch}} = \hat{\omega}_{\text{ch}}$, $0 = N'_{\text{ch}} < N''_{\text{ch}} < \infty$,

$$L'_{\text{chN}} = H_{\text{ch}} - L'_{\text{chT}} \text{ and } L''_{\text{chN}} = \left[\left(\frac{1 - \alpha_{\text{ch}}}{1 - \gamma_{\text{ch}}} \right)^{\eta_{\text{ch}}} \frac{N''_{\text{ch}} (L''_{\text{chT}})^{1 - \alpha_{\text{ch}}(1 - \eta_{\text{ch}})}}{\alpha_{\text{ch}} D_{\text{ch}}} \right]^{\frac{1}{\gamma_{\text{ch}} \eta_{\text{ch}}}} < H_{\text{ch}} - L''_{\text{chT}}.$$

Notice that, if all other government policies are the same ($\bar{g}_{\text{ch}} = \hat{g}_{\text{ch}}$ and $\bar{\omega}_{\text{ch}} = \hat{\omega}_{\text{ch}}$), China's asymptotic equilibria are the same under both exchange-rate regimes. Hence, China's long-run growth is not affected by the exchange-rate regime chosen by the Chinese authorities.

5 Proof of Proposition 3

We know from paragraph 4.1 that $\lim_{t \rightarrow \infty} \rho_{\text{GDP}_{\text{cht}}} = \lim_{t \rightarrow \infty} \rho_{\text{cht}} = \rho_{\text{ch}}$ and that $\frac{\partial \rho_{\text{ch}}}{\partial L_{\text{chT}}} > 0$. Thus, we have to

$$\text{show that } \frac{\partial L'_{\text{chT}}}{\partial g_{\text{ch}}} \begin{cases} > \\ = \\ < \end{cases} 0 \text{ whenever } \zeta_{\text{ch}} \begin{cases} < \\ = \\ > \end{cases} \bar{\zeta}_{\text{ch}}, \text{ where } g_{\text{ch}} = \bar{g}_{\text{ch}} \text{ (or } g_{\text{ch}} = \hat{g}_{\text{ch}}).$$

As $L_{\text{chN}} = H_{\text{ch}} - L_{\text{chT}}$ and $g_{\text{ch}} = \bar{g}_{\text{ch}}$ (or $g_{\text{ch}} = \hat{g}_{\text{ch}}$), equation (A56) (or equation (A57)) can be rewritten as

$$\begin{aligned} w(L_{\text{chT}}, H_{\text{ch}}, \zeta_{\text{ch}}, g_{\text{ch}}) &= \frac{\left[1 + \frac{(1 - \gamma_{\text{ch}}) \alpha_{\text{ch}} (H_{\text{ch}} - L_{\text{chT}})}{(1 - \alpha_{\text{ch}}) \gamma_{\text{ch}} L_{\text{chT}}} \right]}{[\theta_{\text{ch}}(1 - \alpha_{\text{ch}}) L_{\text{chT}}^{\alpha_{\text{ch}}} - (1 - \delta_{\text{ch}})(1 - \theta_{\text{ch}})]^1} + \frac{(1 - \eta_{\text{ch}}) \alpha_{\text{ch}} (H_{\text{ch}} - L_{\text{chT}})}{\eta_{\text{ch}} \gamma_{\text{ch}} L_{\text{chT}}^{1 - \alpha_{\text{ch}}}} - \\ &- L_{\text{chT}}^{\alpha_{\text{ch}}} - \left[\frac{\left[\frac{(1 - \eta_{\text{ch}})(1 - \alpha_{\text{ch}}) L_{\text{chT}}^{\alpha_{\text{ch}}} \zeta_{\text{ch}}}{\eta_{\text{ch}}(1 - \gamma_{\text{ch}})(H_{\text{ch}} - L_{\text{chT}})^{\gamma_{\text{ch}}}} - 1 \right] \left(\frac{\alpha_{\text{ch}}(H_{\text{ch}} - L_{\text{chT}})}{\gamma_{\text{ch}} L_{\text{chT}}} + 1 \right)}{\left(\frac{(1 - \alpha_{\text{ch}}) L_{\text{chT}}^{\alpha_{\text{ch}}} \zeta_{\text{ch}}}{(1 - \gamma_{\text{ch}})(H_{\text{ch}} - L_{\text{chT}})^{\gamma_{\text{ch}}}} + 1 \right)} \right] L_{\text{chT}}^{\alpha_{\text{ch}}} g_{\text{ch}} = 0. \quad (\text{A58}) \end{aligned}$$

One can check that there is a unique value of L_{chT} satisfying $w(L_{\text{chT}}, H_{\text{ch}}, \zeta_{\text{ch}}, g_{\text{ch}}) = 0$:

$$L'_{\text{chT}} = p(H_{\text{ch}}, \zeta_{\text{ch}}, g_{\text{ch}}). \text{ To check that } \frac{\partial p(\cdot)}{\partial g_{\text{ch}}} \begin{cases} > \\ = \\ < \end{cases} 0 \text{ whenever } \zeta_{\text{ch}} \begin{cases} < \\ = \\ > \end{cases} \bar{\zeta}_{\text{ch}}, \text{ consider } L'_{\text{chT}}|_{g_{\text{ch}}=0} =$$

$= p(H_{\text{ch}}, \zeta_{\text{ch}}, g_{\text{ch}})|_{g_{\text{ch}}=0}$, that is the asymptotic (full-employment) equilibrium level of employment in the

tradable sector of China conditional on $g_{ch}=0$, and define $\bar{\zeta}_{ch} \equiv \frac{\eta_{ch}(1-\gamma_{ch})\left(H_{ch}-L'_{chT}\right|_{g_{ch}=0})^{\gamma_{ch}}}{(1-\eta_{ch})(1-\alpha_{ch})\left(L'_{chT}\right|_{g_{ch}=0})^{\alpha_{ch}}}$. One

can easily verify that

$$\text{if } \zeta_{ch} > \bar{\zeta}_{ch}, \text{ then } \left. \frac{\partial w(.)}{\partial g_{ch}} \right|_{L_{chT} \geq L'_{chT}|_{g_{ch}=0}} < 0 \quad (A59)$$

and

$$\text{if } \zeta_{ch} < \bar{\zeta}_{ch}, \text{ then } \left. \frac{\partial w(.)}{\partial g_{ch}} \right|_{L_{chT} \leq L'_{chT}|_{g_{ch}=0}} > 0, \quad (A60)$$

$$\text{where, } \frac{\partial w(.)}{\partial g_{ch}} = - \left\{ \frac{\left[\frac{(1-\eta_{ch})(1-\alpha_{ch})L_{chT}^{\alpha_{ch}}\zeta_{ch}}{\eta_{ch}(1-\gamma_{ch})(H_{ch}-L_{chT})^{\gamma_{ch}}} - 1 \right] \left(\frac{\alpha_{ch}(H_{ch}-L_{chT})}{\gamma_{ch}L_{chT}} + 1 \right)}{\left(\frac{(1-\alpha_{ch})L_{chT}^{\alpha_{ch}}\zeta_{ch}}{(1-\gamma_{ch})(H_{ch}-L_{chT})^{\gamma_{ch}}} + 1 \right)} \right\} L_{chT}^{\alpha_{ch}}.$$

Moreover, consider that

$$\left. \frac{\partial w(.)}{\partial L_{chT}} \right|_{L_{chT} = L'_{chT}} < 0, \quad (A61)$$

thus implying that in a neighborhood of $L'_{chT}|_{g_{ch}=0}$ one has

$$w(L_{chT}, H_{ch}, \zeta_{ch}, g_{ch})|_{g_{ch}=0} \begin{cases} < \\ = \\ > \end{cases} 0 \text{ whenever } L_{chT} \begin{cases} > \\ = \\ < \end{cases} L'_{chT}|_{g_{ch}=0}, \quad (A62)$$

where $w(L_{chT}, H_{ch}, \zeta_{ch}, g_{ch})|_{g_{ch}=0} =$

$$= \frac{\left[1 + \frac{(1-\gamma_{ch})\alpha_{ch}(H_{ch}-L_{chT})}{(1-\alpha_{ch})\gamma_{ch}L_{chT}} \right]}{[\theta_{ch}(1-\alpha_{ch})L_{chT}^{\alpha_{ch}} - (1-\delta_{ch})(1-\theta_{ch})]^{-1}} + \frac{(1-\eta_{ch})\alpha_{ch}(H_{ch}-L_{chT})}{\eta_{ch}\gamma_{ch}L_{chT}^{1-\alpha_{ch}}} - L_{chT}^{\alpha_{ch}}.$$

From (A59), (A60) and (A62)—together—one can conclude that

$$L'_{chT} = p(H_{ch}, \zeta_{ch}, g_{ch})|_{g_{ch}=0} \begin{cases} < \\ = \\ > \end{cases} L'_{chT}|_{g_{ch}=0} \text{ whenever } \zeta_{ch} \begin{cases} > \\ = \\ < \end{cases} \bar{\zeta}_{ch}. \quad (A63)$$

Furthermore, consider that

$$w(L_{chT}, H_{ch}, \zeta_{ch}, g_{ch}) = w(L_{chT}, H_{ch}, \zeta_{ch}, g_{ch})|_{g_{ch}=0} + g_{ch} \frac{\partial w(.)}{\partial g_{ch}} = 0. \text{ This allows us to conclude from}$$

$$(A61), (A62) \text{ and (A63) that } \frac{\partial p(.)}{\partial g_{ch}} = - \frac{\frac{\partial w(.)}{\partial g_{ch}}}{\frac{\partial w(.)}{\partial L_{chT}}} \begin{cases} < \\ = \\ > \end{cases} 0 \text{ whenever } \zeta_{ch} \begin{cases} > \\ = \\ < \end{cases} \bar{\zeta}_{ch}.$$

Finally, one should observe that the absolute value of $\frac{\partial w(.)}{\partial g_{ch}}$ becomes smaller as ζ_{ch} becomes closer to

$\bar{\zeta}_{ch}$, thus reducing the effect of a change in g_{ch} on the asymptotic rate of real GDP growth.

6 Proof of Proposition 4

Again, we know from paragraph 4.1 that $\lim_{t \rightarrow \infty} \rho_{GDP_{cht}} = \lim_{t \rightarrow \infty} \rho_{cht} = \rho_{ch}$ and that $\frac{\partial \rho_{ch}}{\partial L_{chT}} > 0$. Thus, we

have to show that $\frac{\partial L'_{chT}}{\partial H_{ch}} > 0$, where $L'_{chT} = p(H_{ch}, \zeta_{ch}, g_{ch})$. This can be shown by considering (A61) and

the fact that $\left. \frac{\partial w(.)}{\partial H_{ch}} \right|_{L_{chT} = L'_{chT}} > 0$, where $w(.)$ is given by (A58).

7 Transitional paths of the economy

7.1 Suppose $t^* < \infty$. In a neighborhood of $(L'_{chT}, Z' = 0, N'_{ch} = 0)$, we know that N_{cht} converges monotonically to zero since $\rho'_{ch} > \bar{\omega}_{ch}$, and that N_{cht} does not enter the system (A51)-(A52) governing the dynamics of L_{chTt} and Z_t since $L'_{chN} = H_{ch} - L'_{chT}$. Hence, one can study the dynamics of L_{chTt} and Z_t in a neighborhood of $(L'_{chT}, Z' = 0)$ by linearizing (A51)-(A52) around it. In this way, one can

find the eigenvalues $\kappa_1 = -\frac{\Psi_{L_{chTt}}}{\Psi_{L_{chTt+1}}}$ and $\kappa_2 = -\frac{\Lambda_{Z_t}}{\Lambda_{Z_{t+1}}}$, where $\kappa_1 > 1$ and $0 < \kappa_2 < 1$, since

$-\Psi_{L_{chTt}} > \Psi_{L_{chTt+1}} > 0$ and $-\frac{\Lambda_{Z_t}}{\Lambda_{Z_{t+1}}} = \frac{1 + \rho'_{us}}{1 + \rho'_{ch}}$ (notice that the partial derivatives $\Psi_{L_{chTt}}$, $\Psi_{L_{chTt+1}}$, Λ_{Z_t}

and $\Lambda_{Z_{t+1}}$ are evaluated at $(L'_{chT}, Z' = 0)$). Having only one initial condition (solely $Z_{t'}$ is given at time

t' , that is the period in which the economy enters a neighbourhood of $(L'_{chT}, Z' = 0)$, $\kappa_1 > 1$ and

$0 < \kappa_2 < 1$ imply that the linearized system is saddle-path stable. Hence, the system obtained by

linearizing (A51)-(A52) around $(L'_{chT}, Z' = 0)$ has only one path converging to it, which is governed by

$$\tilde{L}_{chTt} \equiv L_{chTt} - L'_{chT} = \frac{Z_t \Psi_{Z_t}}{(\kappa_1 - \kappa_2) \Psi_{L_{chTt+1}}}, \quad t \geq t', \quad (A64)$$

$$Z_t = Z_{t'} \kappa_2^{t-t'}, \quad t \geq t', \quad (A65)$$

where the eigenvector $\frac{\Psi_{Z_t}}{(\kappa_1 - \kappa_2) \Psi_{L_{chTt+1}}} \begin{cases} > \\ = \\ < \end{cases} 0$ whenever $TA_{us} = \lim_{t \rightarrow \infty} TA_{ust} \begin{cases} < \\ = \\ > \end{cases} 0$ (notice that also the

partial derivative Ψ_{Z_t} is evaluated at $(L'_{chT}, Z' = 0)$).

To check that $\frac{\Psi_{Z_t}}{(\kappa_1 - \kappa_2)\Psi_{L_{chTt+1}}} \begin{cases} > \\ = \\ < \end{cases} 0$ whenever $TA_{us} = \lim_{t \rightarrow \infty} TA_{ust} \begin{cases} < \\ = \\ > \end{cases} 0$, consider that $\kappa_1 > \kappa_2$,

$$\Psi_{L_{chTt+1}} > 0 \text{ and } \Psi_{Z_t} = \frac{-TA_{us}}{P_{usT}K_{usT}} = b(L_{chTt+1} = L'_{chT}, L_{chTt} = L'_{chT}, \bar{g}_{us}) .$$

This implies that—along the transitional path— $L_{chTt} > L'_{chT}$ if and only if $TA_{us} < 0$.

7.2 Suppose again that $t^* < \infty$. In a neighborhood of $(L'_{chT}, Z'' = 0, N''_{ch})$, we know that N_{cht} converges to N''_{ch} , $0 < N''_{ch} < \infty$, since $\rho''_{ch} = \bar{\omega}_{ch}$, and that N_{cht} affects the motion of L_{chTt} and Z_t since it enters equations (A51)-(A52). Hence, one can study the dynamics of L_{chTt} , Z_t and N_{cht} in a neighborhood of $(L'_{chT}, Z'' = 0, N''_{ch})$ by linearizing (A51)-(A53) around it. By solving the characteristic equation of the

linearized system, one can find the eigenvalues $\varphi_1, \varphi_2 = \frac{a_{11} + a_{22}}{2} \pm \sqrt{\left(\frac{a_{11} + a_{22}}{2}\right)^2 - a_{11}a_{22} + a_{12}a_{21}}$ and

$\varphi_3 = a_{33}$, where:

$$a_{11} = \frac{\Psi_{N_{cht+1}} \Lambda_{L_{chTt}} - \Psi_{L_{chTt}} \Lambda_{N_{cht+1}}}{\Psi_{L_{chTt+1}} \Lambda_{N_{chTt+1}} - \Psi_{N_{cht+1}} \Lambda_{L_{chTt+1}}}, a_{12} = \frac{\Psi_{N_{cht+1}} \Lambda_{N_{cht}} - \Psi_{N_{cht}} \Lambda_{N_{cht+1}}}{\Psi_{L_{chTt+1}} \Lambda_{N_{chTt+1}} - \Psi_{N_{cht+1}} \Lambda_{L_{chTt+1}}},$$

$$a_{21} = \frac{\Psi_{L_{chTt}} \Lambda_{L_{chTt+1}} - \Psi_{L_{chTt+1}} \Lambda_{L_{chTt}}}{\Psi_{L_{chTt+1}} \Lambda_{N_{chTt+1}} - \Psi_{N_{cht+1}} \Lambda_{L_{chTt+1}}}, a_{22} = \frac{\Psi_{N_{cht}} \Lambda_{L_{chTt+1}} - \Psi_{L_{chTt+1}} \Lambda_{N_{cht}}}{\Psi_{L_{chTt+1}} \Lambda_{N_{chTt+1}} - \Psi_{N_{cht+1}} \Lambda_{L_{chTt+1}}} \quad \text{and}$$

$$a_{33} = -\frac{\Theta_{Z_t}}{\Theta_{Z_{t+1}}} \text{ (notice that all derivatives must be evaluated at } (L'_{chT}, Z'' = 0, N''_{ch}) \text{)}. \text{ One can easily check}$$

that $0 < \varphi_3 < 1$ since $-\frac{\Theta_{Z_t}}{\Theta_{Z_{t+1}}} = \frac{(1 + \rho''_{us})}{(1 + \bar{\omega}_{ch})}$. Moreover, for admissible sets of parameter values one can show

that $\varphi_1 > 1$ and $\varphi_2 > 1$ (for instance, setting $\alpha_{ch} = \gamma_{ch} = 2/3$, $\eta_{ch} = 0.5$, $\delta_{ch} = 0.05$, $\theta_{ch} = 0.95$, $\theta_{us} = 0.945$, $H_{ch} = 0.3552635$, $\bar{\omega}_{ch} = 0.01$, $\bar{g}_{ch} = 0$, one obtains: $L'_{chT} = 0.2099999$, $L''_{chT} = 0.1977922$, $Z' = Z'' = 0$, $N'_{ch} = 0$, $N''_{ch} = 0.5078785$, $L'_{chN} = 0.1452636$, $L''_{chN} = 0.1383763$, $\rho'_{ch} = 0.0143789$, $\rho''_{ch} = 0.01$, $\rho''_{us} = 0.0046842$, $\varphi_1 = 1.0715746$, $\varphi_2 = 2.0797334$, $\varphi_3 = 0.9947368$). Since the values of Z_t and N_{cht} are pre-determined at time t^* (that is the period in which the economy enters a neighbourhood of $(L'_{chT}, Z'' = 0, N''_{ch})$), $\varphi_1 > 1$, $\varphi_2 > 1$ and $0 < \varphi_3 < 1$ imply that the linearized system is unstable. In the special case in which the policy makers manage to control the economy so as to have $N_{cht^*} - N''_{ch} = q_{23}Z_{t^*}$, the linearized system can converge to $(L'_{chT}, Z'' = 0, N''_{ch})$ along the path governed by

$$\tilde{L}_{chTt} = q_{13}Z_{t^*}\varphi_3^{t-t^*}, \quad t \geq t^*, \quad (A66)$$

$$\tilde{N}_{cht} \equiv N_{cht} - N''_{ch} = q_{23}Z_{t^*}\varphi_3^{t-t^*}, \quad t \geq t^*, \quad (A67)$$

$$Z_t = Z_{t^*}\varphi_3^{t-t^*}, \quad t \geq t^*, \quad (A68)$$

where the eigenvectors $q_{13} = -\frac{[a_{12}a_{23} + a_{13}(\varphi_3 - a_{22})]}{[a_{12}a_{21} - (\varphi_3 - a_{11})(\varphi_3 - a_{22})]}$ and $q_{23} = -\frac{[a_{13}a_{21} + a_{23}(\varphi_3 - a_{11})]}{[a_{12}a_{21} - (\varphi_3 - a_{11})(\varphi_3 - a_{22})]}$ can

be computed by using $a_{11}, a_{12}, a_{21}, a_{22},$ $a_{13} = \frac{-\Psi_{Z_t} \Lambda_{N_{cht+1}}}{\Psi_{L_{cht+1}} \Lambda_{N_{cht+1}} - \Psi_{N_{cht+1}} \Lambda_{L_{cht+1}}}$ and

$$a_{23} = \frac{\Psi_{Z_t} \Lambda_{L_{cht+1}}}{\Psi_{L_{cht+1}} \Lambda_{N_{cht+1}} - \Psi_{N_{cht+1}} \Lambda_{L_{cht+1}}}.$$

7.3 Suppose now that $t^* \rightarrow \infty$ and $Q > \underline{Q}_t \quad \forall t > 0$. In a neighborhood of $(L'_{chT}, Z' = 0, N'_{ch} = 0)$, we

know that N_{cht} converges monotonically to zero since $\rho'_{ch} > \hat{\omega}_{ch}$ and that N_{cht} does not enter the system (A45)-(A46) governing the dynamics of L_{chTt} and Z_t since $L'_{chN} = H_{ch} - L'_{chT}$. Hence, one can study the dynamics of L_{chTt} and Z_t in a neighborhood of $(L'_{chT}, Z' = 0)$ by linearizing (A45)-(A46)

around it. In this way, one can find the eigenvalues $\beta_1 = -\frac{\Omega_{L_{chTt}}}{\Omega_{L_{cht+1}}}$ and $\beta_2 = -\frac{\Phi_{Z_t}}{\Phi_{Z_{t+1}}}$, where $\beta_1 > 1$ and

$0 < \beta_2 < 1$, since $-\Omega_{L_{chTt}} > \Omega_{L_{cht+1}} > 0$ and $-\frac{\Phi_{Z_t}}{\Phi_{Z_{t+1}}} = \frac{1 + \rho'_{us}}{1 + \rho'_{ch}}$ (notice that the partial derivatives $\Omega_{L_{chTt}},$

$\Omega_{L_{cht+1}}, \Phi_{Z_t}$ and $\Phi_{Z_{t+1}}$ are evaluated at $(L'_{chT}, Z' = 0)$). Having only one initial condition (solely $Z_{t'}$ is given at time t'), $\beta_1 > 1$ and $0 < \beta_2 < 1$ imply that the linearized system is saddle-path stable. Hence, the system obtained by linearizing (A45)-(A46) around $(L'_{chT}, Z' = 0)$ has only one path converging to it, which is governed by

$$\tilde{L}_{chTt} = \frac{Z_t \Omega_{Z_t}}{(\beta_1 - \beta_2) \Omega_{L_{cht+1}}}, \quad t \geq t', \quad (A69)$$

$$Z_t = Z_{t'} \beta_2^{t-t'}, \quad t \geq t', \quad (A70)$$

where the eigenvector $\frac{\Omega_{Z_t}}{(\beta_1 - \beta_2) \Omega_{L_{cht+1}}} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$ whenever $TA_{us} = \lim_{t \rightarrow \infty} TA_{ust} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0$ (notice that also the

partial derivative Ω_{Z_t} is evaluated at $(L'_{chT}, Z' = 0)$. To check that $\frac{\Omega_{Z_t}}{(\beta_1 - \beta_2) \Omega_{L_{cht+1}}} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$ whenever

$$TA_{us} = \lim_{t \rightarrow \infty} TA_{ust} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0, \text{ consider that } \beta_1 - \beta_2 > 0, \Omega_{L_{cht+1}} > 0 \text{ and } \Omega_{Z_t} = \frac{-TA_{us}}{K_{usT} P_{usT}} =$$

$= n(L_{cht+1} = L'_{chT}, N_{cht+1} = N'_{ch}, L_{chTt} = L'_{chT}, N_{cht} = N'_{ch}, \hat{g}_{us}, \hat{g}_{ch}, Q)$. This implies that—along the transitional path— $L_{chTt} > L'_{chT}$ if and only if $TA_{us} < 0$.

7.4 Suppose again that $t^* \rightarrow \infty$ and $Q > \underline{Q}_t \quad \forall t > 0$. In a neighborhood of $(L''_{cht}, Z'' = 0, N''_{ch})$, we know that N_{cht} converges to N''_{ch} , $0 < N''_{ch} < \infty$, since $\rho''_{ch} = \hat{\omega}_{ch}$, and that N_{cht} affects the motion of L_{cht} and Z_t since it enters equations (A45)-(A46). Hence, one can study the dynamics of L_{cht} , Z_t and N_{cht} in a neighborhood of $(L''_{cht}, Z'' = 0, N''_{ch})$ by linearizing (A45)-(A47) around it. By solving the characteristic equation of the linearized system, one can find the eigenvalues

$$\phi_1, \phi_2 = \frac{p_{11} + p_{22}}{2} \pm \sqrt{\left(\frac{p_{11} + p_{22}}{2}\right)^2 - p_{11}p_{22} + p_{12}p_{21}} \quad \text{and} \quad \phi_3 = p_{33}, \text{ where:}$$

$$p_{11} = \frac{\Omega_{N_{cht+1}} \Phi_{L_{cht}} - \Omega_{L_{cht}} \Phi_{N_{cht+1}}}{\Omega_{L_{cht+1}} \Phi_{N_{cht+1}} - \Omega_{N_{cht+1}} \Phi_{L_{cht+1}}}, \quad p_{12} = \frac{\Omega_{N_{cht+1}} \Phi_{N_{cht}} - \Omega_{N_{cht}} \Phi_{N_{cht+1}}}{\Omega_{L_{cht+1}} \Phi_{N_{cht+1}} - \Omega_{N_{cht+1}} \Phi_{L_{cht+1}}},$$

$$p_{21} = \frac{\Omega_{L_{cht}} \Phi_{L_{cht+1}} - \Omega_{L_{cht+1}} \Phi_{L_{cht}}}{\Omega_{L_{cht+1}} \Phi_{N_{cht+1}} - \Omega_{N_{cht+1}} \Phi_{L_{cht+1}}}, \quad p_{22} = \frac{\Omega_{N_{cht}} \Phi_{L_{cht+1}} - \Omega_{L_{cht+1}} \Phi_{N_{cht}}}{\Omega_{L_{cht+1}} \Phi_{N_{cht+1}} - \Omega_{N_{cht+1}} \Phi_{L_{cht+1}}} \quad \text{and}$$

$$p_{33} = -\frac{\Theta_{Z_t}}{\Theta_{Z_{t+1}}} \text{ (notice that all derivatives must be evaluated at } (L''_{cht}, Z'' = 0, N''_{ch}) \text{)}. \text{ One can easily check}$$

that $0 < \phi_3 < 1$ since $-\frac{\Theta_{Z_t}}{\Theta_{Z_{t+1}}} = \frac{(1 + \rho''_{us})}{(1 + \hat{\omega}_{ch})}$. Moreover, for admissible sets of parameter values one can show

that $\phi_1 > 1$ and $\phi_2 > 1$ (for instance, setting $\alpha_{ch} = \gamma_{ch} = 2/3$, $\eta_{ch} = 0.5$, $\delta_{ch} = 0.05$, $\theta_{ch} = 0.95$, $\theta_{us} = 0.945$, $H_{ch} = 0.3552635$, $\hat{\omega}_{ch} = 0.01$, $\hat{g}_{ch} = 0$, one obtains: $L'_{cht} = 0.2099999$, $L''_{cht} = 0.1977922$, $Z' = Z'' = 0$, $N'_{ch} = 0$, $N''_{ch} = 0.5078785$, $L'_{chN} = 0.1452636$, $L''_{chN} = 0.1383763$, $\rho'_{ch} = 0.0143789$, $\rho''_{ch} = 0.01$, $\phi_1 = 1.0715746$, $\phi_2 = 2.0797334$, $\phi_3 = 0.9947368$). Since the values of Z_t and N_{cht} are pre-determined at time t'' , $\phi_1 > 1$, $\phi_2 > 1$ and $0 < \phi_3 < 1$ imply that the linearized system is unstable. In the special case in which the policy makers manage to control the economy so as to have $N_{cht''} - N''_{ch} = z_{23}Z_{t''}$, the linearized system can converge to $(L''_{cht}, Z'' = 0, N''_{ch})$ along the path governed by

$$\tilde{L}_{cht} = z_{13}Z_{t''}\phi_3^{t-t''}, \quad t \geq t'', \quad (A71)$$

$$\tilde{N}_{cht} \equiv N_{cht} - N''_{ch} = z_{23}Z_{t''}\phi_3^{t-t''}, \quad t \geq t'', \quad (A72)$$

$$Z_t = Z_{t''}\phi_3^{t-t''}, \quad t \geq t'', \quad (A73)$$

where the eigenvectors $z_{13} = -\frac{[p_{12}p_{23} + p_{13}(\phi_3 - p_{22})]}{[p_{12}p_{21} - (\phi_3 - p_{11})(\phi_3 - p_{22})]}$ and $z_{23} = -\frac{[p_{13}p_{21} + p_{23}(\phi_3 - p_{11})]}{[p_{12}p_{21} - (\phi_3 - p_{11})(\phi_3 - p_{22})]}$ can be

computed by using p_{11} , p_{12} , p_{21} , p_{22} ,

$$p_{13} = \frac{-\Omega_{Z_t} \Phi_{N_{cht+1}}}{\Omega_{L_{cht+1}} \Phi_{N_{cht+1}} - \Omega_{N_{cht+1}} \Phi_{L_{cht+1}}} \quad \text{and}$$

$$p_{23} = -\frac{\Phi_{L_{cht+1}}}{\Phi_{N_{cht+1}}} p_{13}.$$

8 Proof of Proposition 6

We can verify in the previous section (7.1. and 7.3, and in particular equations (A64)-(A65) and (A69)-(A70)) that the transitional path converging to $(L'_{chT}, Z'=0)$ depends on the US asymptotic rate of growth ρ'_{us} , which through κ_2 (or β_2) affects the speed at which L_{chTt} and Z_t converge to their respective asymptotic values, and on the US trade account, which through Ψ_{Z_t} (or Φ_{Z_t}) affects in any period the distance of L_{chTt} from its asymptotic value.

Moreover, we know from section 4 (4.1 and 4.2) that along a full-employment equilibrium path the US asymptotic rate of growth is given by $\rho'_{us} = \theta_{us} \left\{ (1 - \alpha_{us}) [l(L'_{chT})]^{\alpha_{us}} + 1 - \delta_{us} \right\} - 1$ (floating exchange-rate) or by

$$\rho'_{us} = \theta_{us} \left\{ (1 - \alpha_{us}) [f(L'_{chT}, m(N'_{ch}, L'_{chT}), \hat{g}_{ch}, \hat{g}_{us}, Q)]^{\alpha_{us}} + 1 - \delta_{us} \right\} - 1 \quad (\text{exchange-rate pegging}).$$

Considering (38) and knowing that $Q > Q_t \forall t$, we can easily check that—other things being equal (in particular if $\hat{g}_{ch} = \bar{g}_{ch}$ and $\hat{g}_{us} = \bar{g}_{us}$)—the US asymptotic rate of growth is higher under floating exchange rate (a regime switch from pegging to floating exchange rate tends to increase the US long-term rate of growth), and that—in case of pegging—a more aggressive “mercantilist” policy on the part of the Chinese authorities (a larger Q) decreases US asymptotic growth.

Finally, one can check that $\Psi_{Z_t} = w(L'_{usT}, H_{us}, \zeta_{us}, \bar{g}_{us})$ (or that $\Omega_{Z_t} = w(L'_{usT}, H_{us}, \zeta_{us}, \hat{g}_{us})$), where

the function $w(\cdot)$ is given by (A58). Since $\left. \frac{\partial w(\cdot)}{\partial L_{usT}} \right|_{L_{usT} = L'_{usT}} < 0$, one can conclude that—other things

being equal (in particular if $\hat{g}_{ch} = \bar{g}_{ch}$ and $\hat{g}_{us} = \bar{g}_{us}$)—a regime switch from pegging to floating exchange rate tends to reduce the US long-term trade deficit, and that—in case of pegging—a more aggressive “mercantilist” policy on the part of the Chinese authorities (a larger Q) tends to increase the US long-term trade deficit.

9 Numerical example comparing the equilibrium values associated with $t^*=1$ to those associated with $t^* \rightarrow \infty$

Let us assume that $\alpha_{ch} = \gamma_{ch} = \alpha_{us} = \gamma_{us} = 2/3$, $\eta_{ch} = \eta_{us} = 0.5$, $\delta_{ch} = \delta_{us} = 0.05$, $\theta_{ch} = 0.95$, $\theta_{us} = 0.945$, $H_{ch} = H_{us} = 0.3552635$, $\hat{\omega}_{ch} = \bar{\omega}_{ch} = 0.001$, $\hat{g}_{ch} = \bar{g}_{ch} = 0$, $\hat{g}_{us} = \bar{g}_{us} = 0.2726248$, $\zeta_{us} = 0.7$, $Q = 0.6990313$, $Z_0 = 0.201173$, $\frac{K_{chN0}}{K_{chT0}} = 0.6969669$, $\frac{K_{usN0}}{K_{usT0}} = 0.6552243$ and $N_{ch0} = 0.4985643$.

As $t^*=1$, one obtains: $L'_{chT} = 0.2099999$, $L'_{chN} = H_{ch} - L'_{chT} = 0.1452636$, $Z' = N'_{ch} = 0$, $\rho'_{ch} = \rho'_{GDP_{ch}} = 0.014379$, $\rho'_{us} = \rho'_{GDP_{us}} = 0.0090401$, $\Psi_{Z_t} = w(l(L'_{chT}), H_{us}, \zeta_{us}, \bar{g}_{us}) = 0$

(which, recalling equation (A64), entails $L_{chTt} = L'_{chT} = 0.2099999$, $L_{chNt} = L'_{chN} = 0.1452636$, $L_{usTt} = L'_{usT} = 0.2099999$ and $L_{usNt} = L'_{usN} = 0.1452636 \quad \forall t \geq 1$), $L_{chT0} = 0.209336$,

$L_{chN0}=0.1444567$ (notice that $L_{chT0}+L_{chN0}<H_{ch}$), $\rho_{ch0}=0.0098086$, $\rho_{GDP_{ch0}}=0.0095518$,

$L_{usT0}=0.2073059$ and $\rho_{us0}=0.024911$.

As $t^*\rightarrow\infty$, we get: $L'_{chT}=0.2099999$, $L'_{chN}=H_{ch}-L'_{chT}=0.1452636$, $Z'=N'_{ch}=0$,

$\rho'_{ch}=\rho'_{GDP_{ch}}=0.014379$, $\rho'_{us}=\rho'_{GDP_{us}}=0.0085096$,

$\Omega_{Z_t}=w(f(L'_{chT},m(N'_{ch},L'_{chT}),\hat{g}_{ch},\hat{g}_{us},Q),H_{us},\zeta_{us},\hat{g}_{us})=0.0055809056$ (which entails—see

equation (A69)—that $L_{chTt}>L'_{chT}=0.2099999$ and $L_{chNt}<L'_{chN}=0.1452636 \quad \forall t \geq 1$). In

particular, along the transitional path converging to (L'_{chT},Z') , one has:

$$L_{chTt}=0.2099999+\left(\frac{3.5434439}{10000}\right)(0.9942138)^{t-1} \quad \forall t \geq 1, \text{ which gives } L_{chT1}=0.2103542,$$

$L_{chN1}=H_{ch}-L_{chT1}=0.1449092$ and $L_{usT1}=0.2087254$, where $Z_1=0.2$, $\beta_1=1.4463793$, $\beta_2=0.9942138$

and $\Omega_{L_{chTt+1}}=6.9664525$. In the initial period, we have: $L_{chT0}=0.2092422$, $L_{chN0}=0.1445215$ (notice

again that $L_{chT0}+L_{chN0}<H_{ch}$), $\rho_{ch0}=0.0135796$, $\rho_{GDP_{ch0}}=0.0109068$, $L_{usT0}=0.2072495$ and

$\rho_{us0}=0.0076692$.

One can easily check that, at time 0, total employment in the market sectors of the economy is larger when $t^*=1$ than when $t^*\rightarrow\infty$, while the opposite is true for the rate of capital accumulation and the rate of real GDP growth. One may also observe that, at time 0, the US rate of capital investment is much larger when $t^*=1$ than when $t^*\rightarrow\infty$.

ⁱ Along an equilibrium path, the real rate of interest, $r_{jt} \equiv \frac{(1+i_{jt})P_{jt-1}}{P_{jt}} - 1$, is given by

$$r_{jt} = \begin{cases} \left(\frac{L_{jTt-1}^{\alpha_j} L_{jNt}^{\gamma_j}}{L_{jTt}^{\alpha_j} L_{jNt-1}^{\gamma_j}} \right)^{\eta_j} \frac{C_{jTt}}{\theta_j C_{jTt-1}} - 1 & \text{if } t > 0 \\ r_{j0} & \text{otherwise, } r_{j0} \text{ given.} \end{cases}$$